We study optimal monetary policy in a New-Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with a credit channel and relationship lending in banking. We show that borrowers’ bank-specific (deep) habits give rise to countercyclical credit spreads, which, in turn, make optimal monetary policy depart substantially from price stability, under both discretion and commitment. Our analysis shows that the welfare costs of setting monetary policy under discretion (with respect to the optimal Ramsey plan) and of using simpler suboptimal policy rules are strictly increasing in the magnitude of deep habits in credit markets and market power in banking.

**JEL codes:** E32, E44, E50

**Keywords:** optimal monetary policy, cost channel, New-Keynesian model, credit frictions, deep habits, credit spreads.

The fact that central banks around the world tend to react sharply to financial shocks and distress in credit markets is well known. However, there is still no consensus on whether and if so, how, monetary policy rules of the Taylor type should respond to financial indicators.

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1. The most important examples are the policy response in the United States during the credit crunch of the late 1980s and, more recently, the Federal Reserve bond-buying programs after the subprime mortgage crisis of 2008.
The main goal of our paper is to enhance our understanding of how credit market imperfections undermine the effectiveness of conventional monetary policy actions, and how the latter should be adjusted to account for credit market considerations. We propose a channel that is capable of generating a positive spread between the lending and the policy rate that has to do with the well-documented existence of imperfect competition in banking\(^2\) and long-term relationships (i.e., customer-market features)\(^3\) in credit markets.

More specifically, we study the conduct of optimal monetary policy in a small-scale New-Keynesian (NK) DSGE framework characterized by monopolistic competition and forward-looking customer markets for loans through deep habits in banking, along the lines of Aliaga-Díaz and Olivero (2010a). Deep habits can be interpreted as representing the existence of switching costs for borrowers, and therefore to capture the documented borrower “hold-up” problem in a parsimonious way.\(^4\) Under deep habits, monopolistically competitive banks set lending rates in a forward-looking fashion: they internalize the fact that, due to habits in banking, current interest rates also affect the future demand for loans by financially constrained firms.

In particular and consistent with the empirical evidence, deep habits generate endogenously countercyclical spreads between the interest rates on loans and the rate on deposits (which, in our model, corresponds to the policy rate). The intuition is as follows: consider a positive aggregate shock that induces an increase in the demand for credit. Having to set the new optimal credit spread, banks will face the following trade-off. On the one hand, they might consider raising it to increase current profits (the “harvesting effect” in the language of the industrial organization literature). On the other hand, they might prefer to lower it as cheaper credit might attract more borrowing firms, which, in turn, will allow banks to generate higher future profits by raising the spread on a locked-in customer base (the “investment effect”).\(^5\) Under a persistent aggregate shock, the “investment effect” dominates, leading to lower credit spreads in good times.

Within this setup, we study the conduct of optimal monetary policy, under both discretion and commitment, and we are able to draw three main conclusions. First, we show that the introduction of deep habits exacerbates the trade-off between stabilizing inflation and the output gap in the face of shocks to total factor productivity (TFP) and the credit spread: the stronger the degree of deep habits, the larger the departure

\(^2\) For evidence on product differentiation as one source of market power in banking, see Kim, Kristiansen, and Vale (2005), Northcott (2004), and Cohen and Mazzeo (2007), among others. For customer switching costs, see Olivero and Yuan (2011), and Kim, Kliger and Vale (2003).


\(^4\) The borrower “hold-up” problem can be rationalized in a context of asymmetric information between lenders and borrowers on borrowers creditworthiness. In this context, incumbent banks gradually accumulate this information as they lend repeatedly, eventually earning, an informational monopoly. This creates switching costs since it is costly for borrowers to switch lenders and to start signaling private information on their creditworthiness to a new bank. Deep habits in credit markets provide a way to model the existence of these costs in a tractable way, without the need to explicitly model information frictions, which is beyond our scope.

\(^5\) See, for instance, the customer-market model of Phelps and Winter (1970).
of the economy from full price stability. With marginal costs directly affected by lending rates, stronger deep habits combined with lower competition in banking create a larger discrepancy between the flexible-price and the efficient levels of output. Keeping inflation at target at all times would lead output to its flexible price level, but not to efficiency. The welfare-relevant output gap would, in fact, respond to movements in the policy rate (which, with output at its flexible price level, would track the natural rate of interest and therefore respond to demand-side shocks), as well as in the credit spread (coming from the endogenous countercyclicality and/or shocks to banking competition).

Second, we highlight the increased importance of optimal monetary policy commitment when there are imperfections in financial intermediation. The welfare gains from committing to a Ramsey plan appear to be quite sizable and to be strictly increasing in the degree of market power and deep habits in banking. As shown in the paper, the presence of deep habits strengthens the role of private expectations for equilibrium outcomes, and therefore gives more power to optimal policy under commitment. In particular, the key distortionary element in the model (the credit spread) is driven not only by current economic conditions—for example, current policy rates, current economic activity (loan demand)—but also (and most importantly) by market expectations on future values for loans demand, policy rates, inflation, and the spread itself. A discretionary policymaker taking those expectations as given is less capable of shielding the economy from exogenous shocks that, through their impact on the credit spread, will destabilize inflation and the output gap. By announcing a credible state-contingent Ramsey plan, a committed policymaker can shape those expectations, and to a certain extent indirectly control fluctuations in credit spreads.

Third, we shed light on the perils of trying to implement the optimal monetary policy plan through simpler interest rate rules. These suboptimal rules generate sizable welfare costs that are increasing in the degree of habits. The underlying motivation has to do with what just discussed: fixed rules do not allow the policymaker to take advantage of its indirect influence on credit spreads via its power to shape private expectations.

Following this introduction the paper is structured as follows: Section 1 presents the model, and Section 2 introduces the semisymmetric equilibrium that we study. Sections 3 discusses alternative frameworks that, in reduced form, are isomorphic to our deep-habits in banking set-up. Section 4 and 5 study the steady state and the aggregate dynamics, respectively. Section 6 contains the results for optimal monetary policy. Section 7 includes a review of related literature and Section 8 concludes. The analytics of the optimal monetary policy problem and some details on the computation of welfare costs are left for the technical appendix.

1. A MODEL WITH DEEP HABITS IN BANKING

We study a closed economy made up of a household sector, a production sector composed of manufacturing and retail firms, a banking sector, and a government.
Households take consumption-saving and labor-leisure decisions to maximize their expected lifetime utility. Manufacturing firms produce intermediate goods with labor as the only input. These firms use a composite of heterogeneous bank loans to finance working capital needs (a fraction of the wage bill has to be paid at the beginning of the period before sales revenues are realized). Banks use households’ savings to provide loans in a monopolistically competitive market. Monopolistically competitive retail firms subject to Calvo-type nominal rigidities produce final consumption goods using intermediate goods. As they are all owned by households, expected future profits made by manufacturing firms, retail firms, and banks will all be discounted using the households’ stochastic discount factor in their respective maximization problem.

The key element of differentiation of our model with respect to a benchmark NK model with a cost channel of monetary policy transmission lies in the assumption that banks provide differentiated loan services to firms and that those services also depend on bank-specific stocks of past loans. This feature—which we refer to as “deep habits” in banking as in Aliaga-Díaz and Olivero (2010a)—is meant to capture the documented existence of long-term lender–borrower relationships in credit markets.

1.1 Manufacturing

A mass one continuum one of perfectly competitive manufacturing firms—each indexed by \( j \in [0, 1] \)—produces an undifferentiated intermediate good using labor \( \tilde{H}_{j,t} \) as the only input, where tildes are used to denote demand. In each period \( t \), the \( j \)th firm sells its output \( I_{j,t} = A_t \tilde{H}_{j,t} \) at the unit price \( Q_{j,t} \) to retailers who use it to produce differentiated final products. The factor \( A_t \) denotes aggregate TFP, which evolves according to the following log-stationary stochastic process:

\[
\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t},
\]

\[
\rho_a \in (0, 1), \quad \varepsilon_{a,t} \sim \text{i.i.d.} \left(0, \sigma_a^2\right).
\]

The firm is subject to a working capital requirement: a fraction \( \alpha \) of labor costs has to be paid before sales revenues are realized. To finance those, the \( j \)th firm uses a composite \( x_{j,t} \) of imperfectly substitutable heterogeneous loans provided by a mass one continuum of banks. This assumption is consistent both with the abundant evidence on the existence of product differentiation in banking (which makes the financial services from different banks imperfectly substitutable from the point of view of firms).
view of borrowers) and with the increasing size of syndicated loans (which account for roughly 50% of originate corporate finance in the U.S.).

Similar to Ravenna and Walsh (2006) and Christiano, Trabandt, and Walentin (2011), we assume that loans are intraperiod, in the sense that they are obtained at the beginning of the period for the firm to meet the working capital requirement, and repaid at the end of the same period. All loans are repaid in full.

In this setup, firms engage in multiple banking relationships by borrowing from several banks in the economy. This is in line with the rich empirical evidence presented by Ongena and Smith (2000a, 2000b). Then, without loss of generality, we assume that each firm borrow from all banks.

To model the existence of borrower “hold-up” effects and costs of switching banks, the loan composite $x_{j,t}$ is assumed to depend also on the accumulated stock of past bank-specific loans as defined by equations (2) and (3):

$$x_{j,t} = \left[ \int_0^1 (l_{jn,t} - \theta s_{n,t-1}) \frac{dn}{\xi_{t-1}} \right]^{\frac{1}{\theta}},$$

$$s_{n,t-1} = \rho_s s_{n,t-2} + (1 - \rho_s) l_{n,t-1},$$

where $l_{jnt}$ is the $j$th firm demand for credit from the $n$th bank in period $t$. The loan composite $x_{j,t}$ is expressed in units of an aggregate consumption good, which will sell at the market price $P_t$. To study the dynamic effects of exogenous shocks to credit, we assume that $\xi_t$, the elasticity of substitution across loan varieties, is stochastic with mean $\xi > 1$, and is generated by the following log-stationary process:

$$\ln \xi_t = (1 - \rho_\xi) \ln \xi + \rho_\xi \ln \xi_{t-1} + \epsilon_{\xi,t} \xi,$$

where $\theta s_{n,t-1}$ in $x_{j,t}$ is intended to capture the borrower “hold-up” effect, with the parameter $\theta$ measuring its extent. We will refer to the case of $\theta > 0$ as “deep habits in banking.” The term $s_{n,t-1}$ in (2) is defined as $s_{n,t-1} \equiv \int_0^1 s_{jn,t-1} dj$, which corresponds to the beginning of period $t$ cross-sectional (across manufacturing firms) average stock of accumulated past loans obtained from the $n$th bank. The fact that $s_{n,t-1}$ is the average (rather than the individual) stock of past borrowing implies that

7. Sufi (2007) provides some discussion on the size of the syndicated loans market in the U.S. For instance, he documents that the average number of lenders in a syndicated loan, in the U.S., is 8.

8. By looking at a sample of more than 1,000 large firms in 20 European countries, these authors find that 50% of firms maintain up to seven bank relationships, while 20% has more than 7. In their sample, the average number of banking relationships is about 6, with Italy and France at the top end with, respectively, 15 and 11. See also references therein for further empirical evidence on multiple banking relationships. Fama (1985), Sharpe (1990), Rajan (1992), Petersen and Rajan (1994), Hart (1995), Von Thadden (1995), Bolton and Scharfstein (1996), Detragiache, Garella, and Giorno (2000), Neuberger, Rathke, and Schacht (2006), and Vulpes (2005) also study various reasons for firms to borrow from multiple banks.

9. This corresponds to the consumer price index (CPI) defined in Section 1.4.
habits are external and are therefore taken as exogenous by each individual borrowing firm.\textsuperscript{10} This simplifying assumption can be rationalized through banks exhibiting economies of scale in the management of informational asymmetries. Thus, the more all firms bank with one bank, the larger the information monopoly for that bank. The stock of habits $s_{n,t-1}$ follows the law of motion in equation (3): it is a linear function of its value in the previous period and the average level of borrowing from the $n$th bank in $t-1$, $l_{n,t-1} = \int_{0}^{1} l_{jn,t-1} dn$.

For $\rho_s = 0$, we have that $s_{n,t-1} = l_{n,t-1}$, so that the deep habits term entering (2) reduces to the last period aggregate amount of loans from the $n$th bank.

In each period $t$, the $j$th firm chooses the level of employment $\bar{H}_{j,t}$, the loans composite $x_{j,t}$, and borrowing $l_{jn,t}$ for $n \in [0,1]$, to maximize the expected present discounted value of its lifetime profits. Its optimization problem is given by

\[
\max_{\{H_{j,t}, x_{j,t}, l_{jn,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \mathcal{F}_{0,t} V_{j,t}^M \\
\text{s.t.} \\
V_{j,t}^M = Q_{j,t} A_t \bar{H}_{j,t} + P_t x_{j,t} - (1 - \tau)W_t \bar{H}_{j,t} - \int_{0}^{1} R_{n,t}^L P_t l_{jn,t} dn, \tag{6}
\]

\[
x_{j,t} = \left[ \int_{0}^{1} (l_{jn,t} - \theta s_{n,t-1}) \frac{l_{jn,t}}{\bar{H}_{j,t}} dn \right] \frac{1}{\bar{H}_{j,t}}, \tag{7}
\]

\[
P_t x_{j,t} \geq \alpha (1 - \tau)W_t \bar{H}_{j,t} \quad \alpha \leq 1, \tag{8}
\]

where $\mathcal{F}_{0,t} \equiv \beta^t (U_{C,t}/U_{C,0})(P_0/P_t)$ is the representative household’s stochastic discount factor.

From the firm’s perspective, the effective cash flow provided by loans is given by the loan composite $x_{j,t}$ and not by the simple integral of loans across banks. Due to imperfect substitutability, each loan provides differentiated liquidity services to the borrowing firm. This is captured by the habit-adjusted Dixit–Stiglitz loan aggregator in (7).\textsuperscript{11} Letting $W_t$ and $R_{n,t}^L$ denote, respectively, the nominal wage rate and the gross interest rate contracted with the $n$th bank, equation (6) defines then the $j$th firm’s cash flow $V_{j,t}^M$ in period $t$ as sales revenues plus what the firm obtains from borrowing minus the sum of labor and borrowing costs. As standard in the NK literature, we assume

\[
\text{10. Just as in Ravn, Schmitt-Grohé, and Uribe (2006), this assumption makes the model analytically tractable, since it preserves the separation of the dynamic problem of choosing total borrowing over time from the static problem of choosing individual borrowing from each bank in every period. If this was not the case, the current demand from the $n$-th bank would depend both on its current relative interest rate and on all future expected rates. Therefore, each bank would face an incentive to renege from past interest rate promises, and the problem would no longer be time consistent.}

\[
\text{11. Notice that under perfect substitutability (which, in the nonstochastic case, occurs in the limit case of $\xi \rightarrow \infty$) and without deep habits, the loan aggregator is simply } x_{j,t} = \int_{0}^{1} l_{jn,t} dn.
\]
that a fiscal authority subsidizes labor costs at a rate \( \tau \) to eliminate all distortions in the steady-state equilibrium. The liquidity in advance constraint (8) states that the liquidity services provided by the differentiated loans should be at least equal to a fraction \( \alpha \) of working capital needs (in this case, labor costs).\(^{12}\)

We solve the problem in two steps. First, we find the \( j \)th firm optimal relative demand for loans issued by the \( n \)th bank. This is obtained by minimizing total borrowing costs, \( \int_0^1 R_{L,n,t} l_{jn,t} \, dn \), subject to (7). The solution gives an expression for \( l_{jn,t} \)—the \( j \)th firm optimal demand for loans issued by the \( n \)th bank—as a function of the relative loan rate charged by the \( n \)th bank and the stock of borrowing habits related to the same loan variety:

\[
l_{jn,t} = \left( \frac{R_{L,n,t}}{R_t} \right)^{-\xi_t} x_{jt} + \theta s_{n,t-1},
\]

where \( R_{L,t} \equiv \left[ \int_0^1 (R_{L,n,t})^{1-\xi_t} \, dn \right]^{1\over\xi_t} \) defines the aggregate loan rate index. As equation (9) states, \( l_{jn,t} \) is higher, the cheaper is borrowing from the \( n \)th bank (i.e., lower \( R_{L,n,t} / R_{L,t} \)) and/or the stronger the lender–borrower relationship established with that bank (i.e., larger \( \theta \) and/or \( s_{n,t-1} \)).\(^{13}\)

By simple calculus, we can derive an expression for the (negative) interest rate elasticity of the demand for loans as:

\[
- \frac{\partial l_{jn,t}}{\partial R_{L,n,t}} \frac{R_{L,n,t}}{l_{jn,t}} = \xi_t \left( 1 - \frac{\theta}{\gamma_{jn,t}} \right),
\]

where \( \gamma_{jn,t} \equiv l_{jn,t} / s_{n,t-1} \) can be interpreted as the growth rate of loans obtained from the \( n \)th bank with respect to the initial stock of loan habits. By setting \( \theta = 0 \) in (9) and (10), we can show that, without deep habits in banking, the model boils down to a benchmark version where the demand for a specific loan variety depends only on the relative interest rate, and the interest rate elasticity of the demand for credit is entirely pinned down by the elasticity of substitution across varieties \( \xi_t \). On the contrary, since the first derivative of the right-hand side of (10) with respect to \( \gamma_{jn,t} \) is positive for \( \theta > 0 \), the interest rate elasticity of the demand for loans appears to be procyclical. This last feature will play a key role in helping our model generate the empirically documented countercyclicality of credit spreads (see Aliaga-Díaz and Olivero 2010b, 2011, Gilchrist and Zakrajsek 2012).

\(^{12}\) For a similar specification of a liquidity in advance constraint with imperfect substitutable assets, see Marimon, Nicolini, and Teles (2012)—who study the competition among different forms of payments—and Végh (2013)—who develops a small open economy model with currency substitution. Canzoneri et al. (2011) is another example of imperfectly substitutable assets in liquidity services (namely, money versus what they refer to as liquid bonds).

\(^{13}\) These two expressions are isomorphic to those found in Ravn, Schmitt-Grohé, and Uribe (2006), with the demand for a specific loan variety, \( l_{n,t} \), and the interest rate index, \( R_{L,t} \), replacing, respectively, the demand for a specific consumption variety and the price index. Habits emerge when the demand for a particular loan is increasing in the stock of habit associated with that variety that requires \( \theta > 0 \).
Using (9) and the definition of the loan rate index $R^L_t$, total borrowing costs entering $V^M_{j,t}$ in equation (6) can be rewritten as $\int_0^1 R^L_{n,t} l_{n,t} d\tau = R^L_t x_{j,t} + \Delta^L_t$, where $\Delta^L_t = \theta \int_0^1 R^L_{n,t} s_{n,t-1} d\tau$ is a term equal across all manufacturing firms. We can then write problem (6) in a simpler form:

$$\max_{H_{j,t}, x_{j,t}} V^M_{j,t} = Q_{j,t} \bar{A}_t \bar{H}_{j,t} + P_t x_{j,t} - (1 - \tau) W_t \bar{H}_{j,t} - P_t \left( R^L_t x_{j,t} + \Delta^L_t \right),$$

s.t.

$$P_t x_{j,t} \geq \alpha (1 - \tau) W_t \bar{H}_{j,t}.$$

Taking first-order conditions with respect to $\bar{H}_{j,t}$ and $x_{j,t}$ and rearranging, we obtain an expression for the optimal pricing of the intermediate goods sold by the $j$th manufacturer:

$$Q_{j,t} = (1 - \tau) \frac{W_t}{A_t} \left[ 1 + \alpha \left( R^L_t - 1 \right) \right] = Q_t. \quad (11)$$

As in Ravenna and Walsh (2006), the working capital requirement implies that prices now also reflect firms’ borrowing costs.

### 1.2 Retailers

A mass one continuum of retail firms, indexed by $i \in [0, 1]$, engages in simple and costless activities such as packaging. They buy the homogeneous intermediate goods from manufacturers to transform it into differentiated final consumption goods for the households. The $i$th retail firm operates in a monopolistically competitive market and produces output $Y_{i,t}$ using the following technology:

$$Y_{i,t} = K \bar{I}_{i,t}, \quad (12)$$

where $\bar{I}_{i,t}$ is the demand for the intermediate goods by the $i$th firm and $K$ is a constant factor of transformation. Notice that $I_{i,t} = I_{j,t}$ since each retailer $i$ is randomly matched to one manufacturer $j$. The $i$th retail firm sells its output at a price $P_{i,t}$ per unit, facing a standard downward-sloping demand: that is, $Y_{i,t} = (P_{i,t} / P_t)^{-\epsilon} Y_t$, where $P_t$ is the economy-wide price index (to be defined in a later section), $Y_t$ is aggregate demand, and $\epsilon > 1$ is the (constant) elasticity of substitution across differentiated final consumption goods. Retail firms are subject to Calvo-type nominal price rigidities: in each period, they face a constant probability $\vartheta$ of not being able to reset their price optimally. Their profit maximization problem is standard:

$$\max_{P_{i,t}} E_0 \sum_{t=0}^\infty \vartheta^t \mathcal{F}_{0,t} V^R_{i,t}$$

s.t. \quad (13)

$$P_{i,t} x_{i,t} \geq \alpha (1 - \tau) W_t \bar{H}_{i,t}. \quad (14)$$
\[ \begin{align*}
V_{i,t}^B &= \left[ (P_{i,t}^* - MC_t) Y_{i,t} \right], \\
Y_{i,t} &= \left( \frac{P_{i,t}^*}{P_t} \right) ^{-\epsilon} Y_t,
\end{align*} \tag{15} \]

where \( MC_t \equiv K^{-1} Q_t \) are nominal marginal costs. After taking first-order conditions and rearranging terms, we obtain the optimal price setting rule for the \( i \)th firm:

\[ \begin{align*}
P_{i,t}^* &= \epsilon \left( E_t \sum_{k=0}^{\infty} \theta^k \beta^k \frac{U_{i,k}^{MC}}{U_{i,k}^{P_t}} e^{\epsilon Y_{t+k}} \right)
\end{align*} \tag{16} \]

1.3 The Banking Sector

There is a mass one continuum of banks indexed by \( n \in [0, 1] \). Each variety of loans/financial services is produced by a bank operating in a monopolistically competitive loan market. Banks are competitive in the market for deposits.

In every period \( t \), the \( n \)th bank chooses its demand for deposits \((D_{n,t})\) and the interest rate on its loans \((R_{L,n,t})\) to maximize the expected present discounted value of lifetime profits. Its optimization problem is given by

\[ \begin{align*}
\max_{\{D_{n,t}, L_{n,t}, R_{L,n,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} F_{0,t} V_{n,t}^B \\
\text{s.t.} \quad V_{n,t}^B &= R_{L,n,t} L_{n,t} - R_t P_t D_{n,t}, \\
L_{n,t} &= D_{n,t} \\
L_{n,t} &= l_{n,t} \equiv \int_0^1 l_{n,t} \, dj = \int_0^1 \left[ \frac{R_{L,n,t}}{R_{L,t}} \right] ^{-\xi_t} \left( \frac{R_{L,n,t}}{R_{L,t}} \right) ^{-\xi_t} x_{jt} + \theta s_{n,t-1} \right] \, dj \\
&= \left( \frac{P_{L,n,t}}{P_{L,t}} \right) ^{-\xi_t} x_t + \theta s_{n,t-1},
\end{align*} \tag{18} \]

where \( x_t \equiv \int_0^1 x_{jt} \, dj \) is the cross-sectional (across manufacturers) average of the demand for the loan composite. The terms \( L_{n,t} \) and \( D_{n,t} \) are measured in units of the aggregate consumption good, and are therefore multiplied by \( P_t \) in (18). Equation (18) defines the bank’s cash flow, where \( R_t \) is the common risk-free gross interest rate on
deposits paid by all banks. Equation (19) defines the bank’s balance sheet, equating loans to deposits (there is no reserve requirement), while equation (20) defines total loans issued by the bank as the sum of the relative demands by all manufacturing firms. Due to loan differentiation and monopolistic competition, the relative loan demand faced by the \( n \)th bank is downward-sloping with respect to \( R_{L,n,t} \), and is internalized by the bank while setting \( R_{L,n,t} \).

The Lagrangian for the \( n \)th bank profit maximization problem is given by:

\[
L = E_0 \sum_{t=0}^{\infty} \mathcal{F}_{0,t} \left\{ (R_{L,n,t} - R_{t}) P_t L_{n,t} + \nu_{n,t} \left[ \left( \frac{R_{L,n,t}}{R^L_t} \right)^{-\xi_t} x_t + \theta s_{n,t-1} - L_{n,t} \right] \right\}.
\]

Taking first-order conditions with respect to \( L_{n,t} \) and \( R_{L,n,t} \) gives, respectively, the following two expressions:

\[
\nu_{n,t} = P_t (R_{L,n,t} - R_{t}) + \theta (1 - \rho_s) E_t (\mathcal{F}_{t,t+1} \nu_{n,t+1}), \tag{21}
\]

\[
L_{n,t} = \xi_t \frac{\nu_{n,t}}{P_t} \frac{x_t}{R^L_t} \left( \frac{R_{L,n,t}}{R^L_t} \right)^{-\xi_t-1}. \tag{22}
\]

Using (20) and (22) to find an expression for \( \nu_{n,t} \)—the shadow value of per unit profits—and substituting the result into (21), after simple algebra, we obtain an expression for the credit spread \( R_{L,n,t} - R_{t} \):

\[
(R_{L,n,t} - R_{t}) = \frac{\gamma_{n,t} \nu_{n,t}}{\xi_t (\gamma_{n,t} - \theta)} - \theta (1 - \rho_s) E_t \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} \nu_{n,t+1}, \tag{23}
\]

where \( \gamma_{n,t} \equiv L_{n,t}/s_{n,t-1} \). Through a close inspection of equation (23), we can see how the introduction of a customer-market channel in banking is capable of generating countercyclical credit spreads. As banks face an increase in current demand conditions (i.e., larger \( \gamma_{n,t} \)) and/or the expected shadow value of future expected profits (i.e., larger \( \nu_{n,t+1} \)), they lower current loan rates to capture a larger customer base that will be locked-in in the future and will allow them to attain higher per unit profits. Another channel, although exogenous, leading to a lower credit spread is related to an increase in the elasticity of substitution across loans \( \xi_t \), which leads to reduced market power in banking.

Notice that if \( \theta = 0 \) (no habits in banking), then equation (23) reduces to the following:

\[
(R_{L,n,t} - R_{t}) = \frac{1}{\xi_t - 1} R_{t}. \tag{24}
\]

If the liquidity services provided by the loan composite do not depend on the past stock of accumulated loans, then the \( n \)th bank sets \( R_{L,n,t} \) as a time-varying markup on
the deposit rate $R_t$ (the marginal cost of issuing one unit of loans to firms). In this case, however, the credit spread might vary only because of exogenous shocks to the elasticity $\xi_t$.

### 1.4 Households

The representative household’s expected lifetime utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_{t+1}^{1-\sigma}}{1 - \sigma} - \frac{H_{t+1}^{1+\phi}}{1 + \phi} \right],$$

(25)

where $H_t = \int_0^1 H_{i,t} dH_i$ are total hours supplied to the manufacturing sector, while $C_t$ is a Dixit–Stiglitz consumption aggregator of a continuum of imperfect substitute final goods, indexed by $i$:

$$C_t = \left[ \int_0^1 C_{i,t} \left( \frac{H_{i,t}}{P_{i,t}} \right)^{1-1/\epsilon} dC_{i,t} \right]^{\epsilon/(\epsilon - 1)}, \quad \epsilon > 1,$$

(26)

where $\epsilon$ denotes the elasticity of substitution across varieties in the goods market.

Given the consumption aggregator (26), the relative consumption demand for the $i$th good is given by:

$$C_{i,t} = \left[ \frac{P_{i,t}}{P_t} \right]^{-\epsilon} C_t,$$

(27)

where $P_t = \left[ \int_0^1 P_{i,t}^{1-\epsilon} dP_{i,t} \right]^{1/(1-\epsilon)}$ is the aggregate CPI, such that $P_t C_t = \int_0^1 P_{i,t} C_{i,t} dP_{i,t}$.

The household is allowed to save by accessing a competitive market for intraperiod bank deposits and by holding money balances. Firms’ and banks’ profits are rebated to households in a lump-sum fashion. The household seeks to maximize (25) subject to the following constraints:

$$M_t + P_t C_t = M_{t-1} + W_t H_t + P_t (R_t - 1) D_t + V_t - P_t T_t,$$

(28)

$$P_t C_t \leq M_{t-1} + W_t H_t - P_t D_t,$$

(29)

where $D_t = \int_0^1 D_{n,t} dn$ are total deposits and $V_t = \int_0^1 V_{M,t}^M dn + \int_0^1 V_{i,t}^R dC_{i,t} + \int_0^1 V_{n,t}^R dn$ are total profits rebated to the household in each period by manufacturers, retailers, and banks. Equation (28) is a standard budget constraint. On its right-hand side, the household’s resources come from previous period money balances, wage income, interest payments on intraperiod deposits and distributed profits net of lump-sum taxes. Equation (29) is the same cash-in-advance (CA) constraint appearing in Ravenna and Walsh (2006): the household’s consumption expenditure cannot exceed money balances accumulated from the previous period, plus wage income, net of resources.
deposited at the banks. This constraint represents the implicit cost of holding intraperiod deposits: money deposited at the bank yields interest, but cannot be used for transaction services. Taking first-order conditions and rearranging, we obtain the following relationships:

\[ C_{t-\sigma} = \beta R_t E_t \left( \frac{C_{t+1}^{\sigma}}{\Pi_{t+1}} \right), \]

\[ H_t^\varphi C_t^\varphi = \frac{W_t}{P_t}, \]

where \( \Pi_{t+1} \equiv P_{t+1}/P_t \) is gross CPI inflation. Equation (30) is a standard Euler equation relating consumption growth to the \textit{ex ante} real interest rate. Equation (31) describes the household’s labor supply schedule.

1.5 The Government

The government is made up of a fiscal and a monetary authority. The fiscal authority sets lump-sum taxes to finance the labor subsidy to firms in the manufacturing sector. It is subject to a balanced-budget rule:

\[ P_t T_t = \tau W_t \tilde{H}_t, \]

where \( \tilde{H}_t = \int_0^1 \tilde{H}_{t,j} dj \) is total labor demand by manufacturers.

Our analysis will mainly focus on optimal monetary policy, whereby the monetary authority sets the short-term riskless interest rate on deposits \( R_t \) in order to maximize aggregate welfare. We will consider both the case of a monetary authority acting under commitment (Ramsey policy) and under discretion (time-consistent policy). We will also compute the welfare costs of adopting simpler but suboptimal policy rules, such as instrumental Taylor rules and policies of both strict and flexible inflation targeting (FIT).

2. EQUILIBRIUM

We consider a semisymmetric equilibrium in the following sense. On the one hand, all banks in the banking sector and all firms in the manufacturing sector will behave identically: that is, banks will set the same interest rate and supply the same amount of loans to all firms, while manufacturers will hire the same amount of labor, produce the same amount of homogeneous intermediate goods, and take on the same amount of loans from banks. On the other hand, due to Calvo contracts, there will be price dispersion in the retail sector: in equilibrium, a fraction \( \vartheta \) of firms will not be able to

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14. Notice that money is the only asset allowing the household to smooth consumption across periods. A CIA constraint is necessary as in its absence, the gross deposit rate (and hence the policy rate too) would be equal to unity in every period. By letting nominal labor income enter the CIA, we rule out a direct effect of interest rate changes on labor supply, as in Ravenna and Walsh (2006).
optimally reset its price, while a fraction \((1 - \theta)\) will. As a consequence, the pricing and production of final goods will differ across retail firms.

Symmetry in the banking sector implies that \(R_{n,t}^L = R_t^L\) and \(L_{n,t} = L_t\) for \(n \in [0, 1]\), while symmetry in the manufacturing sector implies that \(l_{jn,t} = l_{n,t}, \tilde{H}_{jt} = \tilde{H}_t,\) and \(J_{jt} = J_t\) for \(j \in [0, 1]\) (such that the last two equalities also imply that \(I_t = A_t H_t\)). Since market clearing requires that \(L_{n,t} = l_{n,t}\) for \(n \in [0, 1]\), then \(l_{n,t} = l_t = L_t\) as well as \(s_{n,t} = s_t\) for \(n \in [0, 1]\). Using these conditions, from (9), we obtain the following relationship:

\[
x_t = I_t - \theta s_{t-1}. \tag{33}
\]

Under symmetry, using equations (22) and (33), the credit spread equation (23) becomes:

\[
\left( R_t^L - R_t \right) = \frac{R_t^L}{\xi_t} \left( 1 - \frac{\theta}{\gamma_t} \right)^{-1} - \frac{\theta(1 - \rho_s)}{\xi_t} E_t \left[ \mathcal{F}_{t,t+1} \Pi_{t+1} L_{t+1}^L \left( 1 - \frac{\theta}{\gamma_{t+1}} \right)^{-1} \right], \tag{34}
\]

where \(\gamma_t \equiv l_t/s_{t-1}\). Equation (34) further stresses the intertemporal dimension of optimal loan rate setting by banks due to deep habits in credit markets. By forward iteration, it immediately follows that the current rate on bank loans depends both on current as well as expected future market conditions, in particular on expected future policy rates and growth of loan demand.

Real marginal costs faced by retail firms are given by:

\[
m_{C_t} \equiv MC_t = \frac{Q_t}{P_t K}, \tag{35}
\]

where \(Q_t\) has been defined in (11). By market clearing in the final goods market \((Y_{i,t} = C_{i,t} \text{ for } i \in [0, 1]\) such that \(Y_t = C_t\)), the relative demand (27) and technology in the retail sector (12), we have that \(K \tilde{I}_{t,i} = (P_{t,i}/P_t)^{-\epsilon} Y_t\). After integrating both sides across the continuum of \(i\)-indexed retail firms, using the market clearing condition for the intermediate goods market \(\int_0^1 \tilde{I}_{i,t} \, di = I_t\), the technology in the manufacturing sector \(I_t = A_t H_t\), market clearing in the labor market \(\tilde{H}_t = H_t\), and defining the price dispersion index \(\Xi_t \equiv \int_0^1 (P_{t,i}/P_t)^{-\epsilon} \, di\), we obtain the following equilibrium condition:

\[
Y_t / \Xi_t = KA_t H_t. \tag{36}
\]

The household’s labor supply schedule becomes:

\[
H_t^\phi C_t^\sigma = \frac{W_t}{P_t}. \tag{36}
\]

\[15\] Notice that this can also be obtained by imposing symmetry on (2).
Finally, under symmetry, the working capital constraint (8) becomes:

\[ x_t = \alpha (1 - \tau) \frac{W_t}{F_t} H_t. \]  

(37)

3. ALTERNATIVE FRAMEWORKS

We have chosen the deep habits framework as one way to model relationship lending. There are certainly other ways to introduce these “long-lived customer relationships.” In this section, we attempt to draw a link between our model and these alternative ways. First, we can formally show the equivalence between our “deep habits” model and a switching cost model à la Klemperer (1995). For simplicity, let us assume that \( \rho_s = 0 \) so that the stock of habits coincides with the cross-sectional average level of past loans: \( s_{n,t-1} = l_{n,t-1} = L_{n,t-1} \). Moreover, let \( V(L_{n,t-1}) \) be the value of the maximized objective function for the \( n \)th bank for given \( L_{n,t-1} \). Then, we can represent the \( n \)th bank’s optimization problem defined in Section 1.3 in the following recursive form:

\[
V(L_{n,t-1}) = \max_{R_{t-1}^{L_{n,t-1}} L_{n,t}} \left\{ (R_{t-1}^{L_{n,t-1}} - R_t) P_t L_{n,t} + E_{t-1} F_{t+1} V(L_{n,t}) \right\}
\]

s.t.

\[
L_{n,t} = \int_0^1 l_{j,n,t} dj = \int_0^1 \left[ \left( \frac{R_{t-1}^{L_{n,t}}}{R_t} \right)^{-\xi_t} x_{jt} + \theta L_{n,t-1} \right] dj
\]

\[
= \left( \frac{R_{t-1}^{L_{n,t}}}{R_t} \right)^{-\xi_t} x_t + \theta L_{n,t-1}.
\]

Using the constraint to eliminate \( L_{n,t} \), the first-order condition with respect to \( R_{t-1}^{L_{n,t}} \) is:

\[
\frac{\partial \Pi_{n,t}}{\partial R_{t-1}^{L_{n,t}}} + E_t F_{t+1} \frac{\partial V(L_{n,t})}{\partial L_{n,t}} \frac{\partial L_{n,t}}{\partial R_{t-1}^{L_{n,t}}} = 0,
\]

where \( \Pi_{n,t} \equiv (R_{t-1}^{L_{n,t}} - R_t) P_t L_{n,t} \) are temporary profits. This equation is analogous to equation (2) in Klemperer (1995).

We can also draw an equivalence to models with search frictions in physical capital markets as in Kurmann and Petrofsky-Nadeau (2007) and Kurmann (2014). In their environment, households who invest in productive capital meet with firms who need capital for production. In this market, households search for investment opportunities and firms post vacancies for capital. Capital can be in either a productive or a liquid state. Productive capital is rented out to firms at a rate \( \rho \). Liquid capital is made up of new capital made available by households who become capital lenders and
used capital that has been previously separated from other firms. In this context, they denote by $s$ the probability of a match being terminated, in which case a fraction $\psi$ net of depreciation $\delta$ of the capital is returned to the household who receives then $s(1 - \delta)\psi K_t$ units of capital. There are no financial intermediaries in their model and households become direct lenders of capital to firms. Interpreting their $V_K$, that is, their marginal value of a project that is matched and turned into productive capital, as the marginal value of a lender $\nu_n$ in our model, we can then draw an equivalence between their equation for $V_K$ and our condition (21) in Section 1.3 above. In particular,

$$V_K = \frac{\rho_t + \psi (1 - \delta)s}{1 - \theta (1 - \rho_s)}.$$

where $d$ denotes the bank’s net revenue per unit of loans, which can be interpreted as the lending spread, and $(C_{t+1}/C_t)^{\gamma}$ denotes the bank’s owners discount factor given by the household’s intertemporal marginal rate of substitution. This condition is equivalent to equation (21) in our model where the value of the firm is denoted by $\nu_n$, the net revenue is given by the spread $(R_n^L - R)$, and the persistence $(1 - \delta)$ in the second term of the right-hand side is equivalent to $\theta(1 - \rho_s)$. We interpret equation (20) as providing one potential set of microfoundations for the exogenous law of motion for the number of loan contracts $N$ in Stebunovs’ work.

4. STEADY STATE

We focus on a zero inflation ($\Pi_1 = 1$) nonstochastic steady-state equilibrium where $A_t = 1, \xi_t = \xi$, and all remaining variables are constant. Without loss of generality, we set $K = 1$ in the retail sector. First, from the law of motion of real loans (33), we obtain $s = l$ and $x = (1 - \theta)s$. Since $x$ has to be positive, the second equality restricts the deep habits parameter $\theta$ to be smaller than unity.
From the households Euler equation (30), we obtain the steady-state gross interest rate: \( R = \beta^{-1} \). From equation (34), we have that the lending rate \( R^L \) is equal to the riskless rate \( R \) times the constant gross credit markup \( \mu^R \):

\[
R^L = \mu^R R, \tag{38}
\]

where

\[
\mu^R \equiv \frac{m\xi}{m\xi - 1} \quad \text{and} \quad m \equiv \frac{(1 - \theta)}{[1 - \theta\beta(1 - \rho_s)]}. \tag{39}
\]

By imposing an upper bound \( \bar{\theta} < 1 \) on the deep habits parameter \( \theta \), the following assumption guarantees that \( \mu^R > 1 \) (hence, a positive credit spread) at the steady state.

**Assumption 1.** \( \theta < \frac{\xi - 1}{\xi - \beta(1 - \rho_s)} \equiv \bar{\theta} \). \(^{16}\)

Since \( \partial \mu^R / \partial \theta > 0 \), the steady-state markup under deep habits, \( m\xi / (m\xi - 1) \), is always larger than what would obtain under monopolistic competition in banking alone, \( \xi / (\xi - 1) \), for any \( \theta \in (0, \bar{\theta}) \). Moreover, this difference appears to be more significant, the larger the persistence of habits: that is, \( \partial(\partial \mu^R / \partial \theta) / \partial \rho_s > 0 \).

From the optimal price setting by retailers, we have that, at the steady state, real marginal costs are equal to the inverse of the steady-state gross markup in the final goods market, \( \mu \equiv \epsilon / (\epsilon - 1) \):

\[
MC = (1 - \tau) \frac{W}{P} \left[ 1 + \alpha \left( R^L - 1 \right) \right] = \frac{1}{\mu}.
\]

We assume that the fiscal authority sets the subsidy rate \( \tau \) to equate the marginal rate of substitution between consumption and labor to the marginal productivity of labor. This implies setting \( \tau \) to make \( W/P = 1 \), or, more specifically, \( \tau = \{\mu[1 + \alpha(R^L - 1)] - 1\}/\{\mu[1 + \alpha(R^L - 1)]\} \). \(^{17}\) The latter together with the aggregate technology, \( Y = H \), market clearing, \( Y = C \), and the labor supply equation (31) implies that \( Y = 1 \).

**A note on overborrowing:** Equation (8) can be used together with \( l = L \) and equations (3) and (20) to formally show how the presence of deep habits acts as a sort of externality in credit markets. Since banks do not internalize this externality, the equilibrium involves overborrowing as a usual effect of holdup problems. Formally,

\(^{16}\) For \( \theta > \bar{\theta} \), we have that \( \mu^R < 0 \). Assumption 1 easily holds for any realistic parameterization of the degree of imperfect competition in banking, as indexed by \( \xi \). For instance, if \( \rho_s = 0 \) (i.e., the stock of habits is equivalent to the level of past period loans), then \( \bar{\theta} = \frac{\xi - 1}{\xi - \beta} \), which is very close to unity for \( \beta \approx 1 \). If instead \( \rho_s \to 1 \), then \( \bar{\theta} \to (\xi - 1)/\xi \). The latter is also rather close to unity unless one assumes an unrealistically high degree of market power in banking (i.e., low \( \xi \)).

\(^{17}\) This subsidy yields an undistorted steady state that, in turn, allows us to use standard linear-quadratic techniques to compute optimal policies. As discussed in Section 6, for realistic parameterizations of the model, this subsidy is of relatively small magnitude.
steady-state loans are given by:

\[ L = \frac{x}{(1 - \theta)} = \frac{1}{(1 - \theta)} \alpha (1 - \tau) \frac{W}{P} H. \]

In a model without the habits friction, borrowing would be lower by a proportion \((1 - \theta)^{-1}\). Since our environment does not involve capital, we cannot really speak to the problem of under (over) investment typical of environments with holdup problems. However, should we extend the model to allow for physical capital, the overborrowing feature present in our model could be associated with the result of overinvestment of Kurmann (2014).

5. LOG-LINEARIZATION AND AGGREGATE DYNAMICS

We log-linearize the equilibrium conditions around the unique nonstochastic steady state.\(^{18}\) Before doing that, we define \(\tilde{\mu}_t^R \equiv \frac{\hat{R}_t}{R_t}\) as the gross markup in the loan market, such that \(\tilde{\mu}_t^R = \tilde{r}_t^L - \tilde{r}_t\). Henceforth, we will refer to \(\tilde{\mu}_t^R\) as the credit spread. From the households Euler equation (30) and market clearing in the goods market, we obtain:

\[ \hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma} (\tilde{r}_t - E_t \hat{\pi}_{t+1}). \]  

(40)

From the optimal pricing rule of retail firms (17), we obtain the linearized NK Phillips curve:

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{m}c_t, \]  

(41)

where, from the linearization of (11) and (35),

\[ \hat{m}c_t = \hat{w}_t - \hat{p}_t - \hat{a}_t + \eta \hat{r}_t^L, \]  

(42)

with \(\kappa \equiv (1 - \vartheta) (1 - \vartheta \beta)/\vartheta, \eta \equiv \alpha \mu^R / [(1 - \alpha) \beta + \alpha \mu^R]\), and \(\hat{a}_t \equiv \ln A_t\) following the process in (1). The real wage \(\hat{w}_t - \hat{p}_t\) is obtained from the consumption-leisure trade-off condition (36) combined with market clearing, \(C_t = Y_t\), and the aggregate technology, \(Y_t = A_t H_t\):

\[ \hat{w}_t - \hat{p}_t = (\sigma + \varphi) \hat{y}_t - \varphi \hat{a}_t. \]  

(43)

18. Hatted lower case variables denote percentage deviation of variables from their respective steady state.
It follows that the inflation dynamics in our economy are regulated by a Phillips curve augmented with a cost channel and a time-varying credit spread:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\sigma + \varphi) \hat{y}_t + \kappa \eta \hat{r}_t + \kappa \eta \hat{\mu}_R^R - \kappa \varphi \hat{a}_t.
\] (44)

The dynamics of \( \hat{\mu}_R^R \) are then determined by the log-linearized version of (34), describing the optimal interest rate setting in the banking sector. After simple manipulation, we obtain:

\[
\left[1 - \frac{\omega}{1 - \beta \theta (1 - \rho_s)}\right] \hat{\mu}_R^R = \frac{(1 - \theta)^{-1} \theta \omega}{[1 - \beta \theta (1 - \rho_s)]} \left[\theta \beta (1 - \rho_s) E_t \hat{\gamma}_{t+1} - \hat{\gamma}_t\right] (45)
\]

\[= \frac{\theta \beta \omega (1 - \rho_s)}{1 - \beta \theta (1 - \rho_s)} (E_t \hat{r}_{t+1} - \hat{r}_t)
\]

\[= \frac{\theta \beta \omega (1 - \rho_s)}{1 - \beta \theta (1 - \rho_s)} E_t \hat{\mu}_R^R
\]

\[= \frac{\theta \beta \omega (1 - \rho_s)}{1 - \beta \theta (1 - \rho_s)} (\hat{r}_t - E_t \hat{\pi}_{t+1}) - \omega \hat{\xi}_t,
\]

where \( \omega \equiv [1 - \beta \theta (1 - \rho_s)][\xi(1 - \theta)]^{-1}, \hat{\gamma}_t = \hat{l}_t - \hat{s}_{t-1}, \) and \( \hat{\xi}_t \equiv \ln(\xi_t/\xi) \) evolves according to (4). Using the definition of \( \omega \), simple algebra shows that the term within squared brackets multiplying \( \hat{\mu}_R^R \) on the left-hand side of (45) is strictly positive if and only if \( \theta < (\xi - 1)/\xi \). Since \( \omega > 0 \) always, then, as long as \( \theta < (\xi - 1)/\xi \), a negative shock to \( \hat{\xi}_t \) (higher market power in banking) is equivalent to a positive shock to the credit spread. Notice that without deep habits, \( \theta = 0 \), equation (45) reduces to \( \hat{\mu}_R^R = -\hat{\xi}_t (\xi - 1)^{-1} \). In this case, the credit spread \( \hat{\mu}_R^R \) is equivalent to an exogenous cost push shock entering the Phillips curve (44).

A close inspection of (45) shows how the credit spread reacts to different market forces. First of all, since in our model, firms borrow more during upturns, by the first term in squared bracket on the right-hand side, the credit spread decreases during current booms (higher \( \hat{\gamma}_t \)), but increases in response to expected future ones (higher \( E_t \hat{\gamma}_{t+1} \)). Second, it decreases with respect to the expected future change in the deposit rate, \( E_t \hat{r}_{t+1} - \hat{r}_t \): banks would rather wait to charge higher rates on loans if they foresee a higher cost on deposits in the near future. Third, it decreases with respect to expected future spreads, \( E_t \hat{\mu}_R^R \): once a bank anticipates the possibility of charging higher spreads in the future, it would most likely lower current rates to expand its customer base. Fourth, it increases with respect to the real interest rate, \( \hat{r}_t - E_t \hat{\pi}_{t+1} \): a higher real interest rate lowers the present discounted value of future bank’s profit, thus making the building of a future customer base (through lower current rates) less appealing.

19. The inequality \( \theta < (\xi - 1)/\xi \) is stricter than what required by Assumption 1. However, it easily holds for any calibration of \( \xi \) consistent with the average credit spreads observed in the data.
It is instructive to compare our Phillips curve (44) with its counterparts in the baseline Ravenna–Walsh model where firms borrow at the policy rate, as the banking sector is perfectly competitive:

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\sigma + \varphi) \hat{y}_t + \kappa \eta \hat{r}_t - \kappa \psi \hat{a}_t. \]  

(46)

The difference is clear: with \( \hat{r}_L = \hat{r}_t \), there is no credit spread entering the inflation dynamics. With imperfect competition in banking (but no deep habits), a credit spread related term, \( \kappa \eta \hat{\mu}_R \), may enter, as in equation (44). The latter is equal to zero if the degree of imperfect competition is time invariant (i.e., if \( \xi_t = \xi \) in every period), while it fluctuates exogenously if \( \xi_t \) is stochastic as \( \hat{\mu}_R = \hat{\xi}_t (1 - \xi)^{-1} \). With deep habits, \( \hat{\mu}_R \) is determined by (45), that is, it is driven both by exogenous shocks and endogenous market forces.

From the linearization of (33) and (37), and the expression for the real wage in (43), we obtain:

\[ \hat{l}_t = \theta \hat{s}_{t-1} + (1 - \theta) \hat{a}_t, \]  

(47)

where

\[ \hat{a}_t = (1 + \sigma + \varphi) \hat{y}_t - (1 + \varphi) \hat{a}_t. \]  

(48)

After linearizing (3) and using (47), we obtain an expression for the law of motion of the stock of habits:

\[ \hat{s}_t = [\rho_s + (1 - \rho_s) \theta] \hat{s}_{t-1} + (1 - \rho_s) (1 - \theta) \hat{a}_t. \]  

(49)

The model is closed by a path for the nominal interest rate \( \hat{r}_t \) chosen by the monetary authority.

The next section describes the optimal monetary policy plan.

6. OPTIMAL MONETARY POLICY

We study the consequences of deep habits in banking for the design of optimal monetary policy under both discretion and commitment. Before performing such analysis, we define the efficient allocation in our economy. The latter is derived by solving a social planner’s optimization problem, which, given the absence of capital, is equivalent to maximizing the representative agent’s temporary utility subject to the
aggregate technology \( Y_t = A_t H_t \) and market clearing \( Y_t = C_t \):\(^{20}\)

\[
\max \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{H_t^{1+\psi}}{1+\psi}
\]

s.t. \( Y_t = A_t H_t = C_t \).

The solution to this problem gives us \( Y_t^e \), the efficient level of output:

\[
Y_t^e = A_t^{1+\psi}.
\]

As in Ravenna and Walsh (2006), \( Y_t^e \) does not coincide with the flexible price level, \( Y_t^f \). Under flexible prices (i.e., \( \hat{\sigma} = 0 \) in (17)), real marginal costs \( MC_t / P_t \) equal the inverse gross markup \( \frac{1}{\mu} \). Letting \( \hat{R}_L^f \equiv 1 - \alpha + \alpha \hat{R}_L^f \) denote the interest rate distortion per unit of labor, from the household’s labor supply condition and market clearing, it is straightforward to obtain

\[
Y_t^f = \left[ \frac{1}{\mu (1 - \tau) \hat{R}_L^f} \right]^{\frac{1}{1+\psi}} Y_t^e.
\]

Absent the credit distortion (\( \alpha = 0 \)), a constant labor subsidy \( \tau \) such that \( \mu (1 - \tau) = 1 \) would guarantee that \( Y_t^f = Y_t^e \), on and off the steady state. With the credit distortion (\( \alpha \in (0, 1) \)), assuming the government chooses \( \tau = (\mu \hat{R}_L^L - 1) / \mu \hat{R}_L^L \), the equivalence holds only in steady state, otherwise the flexible price level of output deviates from efficiency because of fluctuations in \( \hat{R}_L^f \).\(^{21}\) As a matter of fact, the flexible price level of output can be written as \( Y_t^f = (\hat{R}_L^L / \hat{R}_L^f)^{1+\psi} Y_t^e \). This implies that \( Y_t^f \) might be above (respectively, below) the efficient level when the flexible-price lending rate \( \hat{R}_L^f \) goes below (respectively, above) its steady-state value. The latter is driven by movements in the policy rate \( R_t \) and the credit spread, which come from exogenous variations in the degree of imperfect competition in banking (shocks to \( \xi_t \)) and/or the endogenous borrowing externality (due to deep habits in banking).

The central bank’s objective is obtained by taking a second-order approximation to the representative agent’s welfare. Similar to a benchmark NK model, the objective is expressed in terms of squared deviations of inflation and output from their respective efficient levels:

\[
\mathcal{L}_0 = -\frac{1}{2} \epsilon H_t^{1+\psi} E_0 \sum_0^\infty \beta^t \left[ \dot{\pi}_t^2 + \psi \left( \dot{y}_t^g \right)^2 \right].
\]

\(^{20}\) The social planner overcomes all frictions in the economy, namely, monopolistic competition, the need for cash in transactions, the nominal rigidities in the intermediate goods sector, the working capital needs in manufacturing, and hence the need for financial intermediation.

\(^{21}\) As discussed in Section 4, with \( \tau = (\mu \hat{R}_L^L - 1) / \mu \hat{R}_L^L \), the efficient equilibrium and the equilibrium of the decentralized economy share the same steady state, which is therefore undistorted.
where $H$ is steady-state hours, $\hat{y}_t^f = \hat{y}_t - \hat{y}_t^e$ is the welfare relevant output gap (with $\hat{y}_t^e = \hat{a}_t (1 + \phi)/(\sigma + \varphi)$), and $\psi \equiv (\sigma + \varphi) \kappa \epsilon^{-1}$ is the central bank’s relative concern for output gap versus inflation stabilization in the microfounded loss. Notice that $\psi$ does not depend on structural parameters related to the credit constraint on firms (the share of the wage bill to be paid in advance, $\alpha$) and banking (the average degree of imperfect competition, $\xi$, or the measure of deep habits, $\theta$). Optimal monetary policy requires the maximization of (52) subject to a set of equilibrium constraints given by the aggregate demand equation (30) together with the NK Phillips curve (44), the spread equation (45), the equations for $l$ and $x$, respectively (47) and (48), and the law of motion for $s$ in (49), where output $\hat{y}_t$ is now expressed as $\hat{y}_t = \hat{y}_t^f + \hat{y}_t^e$.

Before getting into qualitative and quantitative optimal monetary policy implications of the model, it is important to highlight the key policy trade-offs/distortions faced by the central bank. It is straightforward to show that, as in the baseline NK model, marginal costs are proportional to the deviation of output from its flexible price level. As a result, from equations (41)–(43) and the log-linear version of (51), the Phillips curve can be written as

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\sigma + \varphi) \left( \hat{y}_t - \hat{y}_t^f \right).$$  \hspace{1cm} (53)

From the latter and the expression for $Y_f^f$ in (51), we can see that, absent credit market distortions (and exogenous cost-push shocks), the divine coincidence would follow: by keeping inflation at target at all times ($\hat{\pi}_t = 0$ for $t \geq 0$), the central bank would also be able to keep output at its flexible-price/efficient level. To make this outcome attainable, the policy rate $\hat{R}_t$ would have to be adjusted by the central bank to insulate output from any demand shock affecting equilibrium outcomes through the Euler equation.

In our model, such divine coincidence breaks. In particular, the efficiency gap $\hat{y}_t - \hat{y}_t^e$ is equal to

$$\hat{y}_t - \hat{y}_t^e = \hat{y}_t - \hat{y}_t^f - \eta \left( \hat{R}_t + \hat{\mu}_t^R \right).$$ \hspace{1cm} (54)

As for the case of no credit market distortion, the Phillips curve (53) still implies that by keeping inflation at target, the central bank can stabilize output at its flexible price level. However, this does not automatically close the efficiency gap, as the latter would still respond to movements in the policy rate $\hat{R}_t$ (which, with output at its flexible price level, would track the natural rate of interest and therefore respond to demand-side shocks hitting the intertemporal Euler equation) and the credit spread $\hat{\mu}_t^R$ (coming from the endogenous countercyclicality due to deep habits and/or the exogenous shocks to banking competition) with elasticity $\eta$.\footnote{The elasticity $\eta$—defined in Section 5—is strictly increasing in $\alpha$ and $\theta$, and strictly decreasing in $\xi$. Hence, the discrepancy between the efficiency and the flexible price gap is more sizable in a model with stronger credit distortions, less competition in banking, and stronger habits in banking.} Alternatively, the central bank could implement a policy keeping the output gap at zero at all times.
However, this would induce fluctuations in marginal costs, which would, in turn, lead to unstable inflation.

Deep habits exacerbate this policy trade-off, thus making full price stability more suboptimal. To see this, let us suppose deep habits were not present. In this case, with $\theta = 0$ in equation (45), the credit spread is simply $\hat{\mu}_R = - (\xi - 1)^{-1} \xi_t$: exogenous variations in banking competition would lead to inefficient fluctuations in output. With deep habits, the trade-off gets further exacerbated. For instance, consider an exogenous decline in $\xi_t$, leading to lower competition in banking. The credit spread would increase, which would, in turn, diminish the demand for loans. The subsequent fall in labor demand and output would generate further hikes in lending rates (via the countercyclicality of spreads), which would then put additional downward pressure on economic activity. The policymaker might then find optimal to give up some price stability in exchange for more contained fluctuations in the output gap.

As an analytical characterization of the optimal plan is unattainable (under both discretion and commitment), we resort to numerical methods and study the model economy’s response to exogenous TFP and credit shocks under both regimes. For this purpose, we adopt the following quarterly calibration. We set the risk aversion parameter $\sigma$ equal to 2 and labor disutility parameter $\varphi$ equal to $1/4$. The latter is consistent with a Frisch elasticity of labor supply equal to 4, as supported by the macro-based labor literature. The subjective discount factor $\beta$ is set to 0.99, which gives a 4% annual real interest rate. For the degree of imperfect competition in the goods market, we choose $\epsilon = 6$, corresponding to a net markup of 20%. For what concerns the degree of price rigidity, we set the Calvo probability of no price change equal to 0.66. This gives an average duration of prices equal to three quarters, consistent with the empirical evidence provided by Nakamura and Steinsson (2010). We set both AR(1) coefficients $\rho_a$ and $\rho_\xi$ equal to 0.9.

To better highlight the quantitative role of deep habits in credit markets, we will consider two alternative parameterizations for the steady-state gross markup in credit markets, $\mu_R$. In the first case, $\mu_R$ is set to give a 2% annual credit spread, as observed in the U.S. We will refer to this case as the case of a “low spread.” In the second case, $\mu_R$ is set to give a 4% annual credit spread. We will refer to this as the case of a “high spread.”

For expositional purposes, we consider two alternative values for the degree of deep habits in credit markets: $\theta = 0$ (no deep habits, which will make our model isomorphic to the cost channel model studied by Ravenna and Walsh 2006, with the shock to banking competition replacing the shock to government spending), and $\theta = 0.5$. There is a couple of reasons for why the range of feasible values for $\theta$ is limited. First, as discussed in Section 4, $\theta$ has to be sufficiently smaller than unity.
(smaller than a certain upper bound $\tilde{\theta}$) to guarantee a positive credit spread in steady state. This is also consistent with the empirical estimates provided by Aliaga-Díaz and Olivero (2010a). Second, it is well-known that, under a cost channel, the NK model is more prone to indeterminacy issues: due to the borrowing constraint faced by firms, the central bank faces a potentially tight upper bound on its responsiveness to inflation in the Taylor rule, as increasing rates does not guarantee lower marginal costs (see Llosa and Tuesta 2009 for a detailed analysis). The presence of deep habits in banking exacerbates this problem. Following a belief-driven surge in inflation, an interest rate hike will drive down output. While this will still lead to lower real wages (as in the baseline NK model without borrowing), it will also increase the spread between the lending rate paid by firms and the policy rate (due to the countercyclicality coming from deep habits). The latter effect will reinforce the direct positive impact on marginal cost coming from the cost channel itself, which will then impose an even tighter upper bound on the policymaker’s response to inflation. For $\theta$ not larger than 0.5, we find equilibrium indeterminacy not to be an issue.

For given parameterization, using the expressions (38) and (39), we will then retrieve the elasticity of substitution across bank loans, indexing the degree of imperfect competition in the banking system that is consistent with each of the two calibrated values of $\mu^R$. Finally, we set $\rho_s$—indexing the persistence of the stock of habits in (3)—equal to zero, such that deep habits are captured by bank-specific lagged aggregate loans. Under the baseline calibration, the labor subsidy $\tau$ needed to induce steady-state efficiency is around 18%.

6.1 The Case of Discretion

Under discretion, we compute the optimal time-consistent monetary policy. For this purpose, we restrict the analysis to the concept of Markov perfect equilibrium, that is, an equilibrium where endogenous variables are functions only of relevant state variables, for example, the outstanding stock of habits $\hat{s}_{t-1}$, and the shocks $\hat{a}_t$ and $\hat{\xi}_t$. Although the lack of commitment implies that policy announcements are not credible, current policy choices can still affect future expectations via their impact on the stock of habits $\hat{s}_t$, a state variable in the next period. Despite the fact that the policymaker cannot strategically exploit this linkage (as it takes it as a given equilibrium relationship), the optimal monetary policy problem is also dynamic under discretion. The latter is an important element of differentiation with respect to the cost channel model of Ravenna and Walsh (2006) for which, because of the absence of endogenous states, the optimal time-consistent policy can be found by solving (analytically) a simple static loss minimization problem. Since this is not possible in our case, we solve for the optimal monetary policy under discretion following the computational procedure proposed by Soderlind (1999).

26. In the analysis, we have experimented positive values for $\rho_s$ finding only very marginal changes in the quantitative results.
27. A similar approach is used in Steinsson (2003) and Leith, Moldovan, and Rossi (2012).
Figure 1 displays the impulse responses to a 1% shock to TFP for key endogenous variables, under three alternative parameterizations: namely, the cost channel model of Ravenna and Walsh (2006) (where \(\alpha = 1\) but \(\theta = 0\): there is a cost channel but deep habits in banking are absent); and two versions of our deep habits economy differing with respect to the size of the steady-state credit spread, as discussed in the previous section.\(^{28}\)

Consider first the case studied by Ravenna and Walsh (2006). Under the optimal policy, the economy displays a positive output gap, a deflation, and a drop in the policy rate. These responses differ from the benchmark NK model without a cost channel where, by lowering the policy rate, the monetary authority is successful at stabilizing both inflation and the output gap at their efficient steady-state level.\(^{29}\) Since with a cost channel movements in the nominal interest rate also affect the credit-related component of marginal costs in the NKPC, the nominal interest rate cannot be used to fully insulate the economy from technology shocks. It therefore appears optimal to generate a larger drop in the policy rate (with respect to the no cost channel case), while letting output and inflation deviate from full efficiency. To capture the underlying mechanism, consider a positive shock to TFP, which, without any policy intervention, would determine a negative output gap. Lowering the nominal interest rate by as much as in the no cost channel NK model might suffice to restore a zero output gap, but at the cost of a deflation as the policy rate (also the lending rate) pulls down the interest rate component of marginal costs in the Phillips curve. To

\(^{28}\) The results for the case of a benchmark NK model where the cost channel is completely absent are already well known, and we do not show them here.

\(^{29}\) The impulse responses of inflation and the output gap would both be flat at zero.
counteract this, optimal policy requires the central bank to lower the nominal rate by a larger amount. By doing this, the central bank induces a positive output gap, which contains the deflation by raising the wage component of marginal costs.

Deep habits amplify the transmission we have just described and determine an even stricter stabilization trade-off for the central bank. As a result, the optimal equilibrium features an even larger output gap, a stronger deflation, and a bigger drop in the policy rate. Key to this mechanism is the countercyclicality of credit spreads generated by deep habits. By creating a positive output gap, the expansionary monetary policy lowers the credit spread, which, in turn, hampers the decline in inflation. The central bank counteracts this channel by lowering the nominal interest rate even further, with the objective of creating an even larger output gap in the attempt to increase the wage component of marginal costs (via a standard aggregate demand channel) and hence contain deflation. As the figure shows, this channel appears to be stronger in an economy where the (average) credit spread is larger. From equation (44) and the definition of the composite parameter $\eta$, it is easy to see that a larger $\mu^R$ makes the credit spread dynamics quantitatively more relevant for inflation determination, leading to an amplification of the cost channel of policy transmission.

Another interesting consequence of deep habits is the hump-shaped response of loans to the TFP shock. Given that output, $\hat{y}_t = \hat{y}_t^g + \hat{y}_t^e$ (not plotted) positively responds on impact to the TFP shock, this result implies that in our model, loans lag output, a feature that is consistent with the empirical evidence provided by Demirel (2014). He finds that business loans are more positively correlated with past than with current output, both in the U.S. and the Euro area, a feature that a standard model of business and credit fluctuations with agency costs cannot generate. While Demirel shows that this could be amended with the introduction of costly financial intermediation, we are able to obtain a similar pattern through deep habits in banking. Figure 2 considers instead the case of a 1% unanticipated shock to the credit spread. Without deep habits, a higher spread generates positive inflation (through a cost-push effect) and a negative output gap (through a negative impact on aggregate activity). To counteract higher prices, the central bank raises the policy rate, which, together with the exogenously driven higher spread, leads to a larger lending rate and a subsequent further pressure on inflation. The positive response of inflation and the negative response of output are exacerbated by the introduction of deep habits: as output declines, the credit spread increases beyond the initial exogenous shock; this leads to a further cost-push increase in inflation and a harsher recession.

30. Other things being equal, the responses to a TFP shock can be amplified by increasing the size of $\theta$. However, as previously discussed, the range of feasible values for $\theta$ is rather limited due to equilibrium indeterminacy issues.

31. This is not limited to the case of optimal policy under discretion, but equally holds under commitment (see below).

32. Since a period in the model corresponds to one quarter, the shock corresponds to a 4% increase in the credit spread at an annual frequency. Given the expression in (45), other things being equal, a 1% positive exogenous shock to the credit spread $\hat{\mu}_t^R$ is equivalent to a $(0.01/\omega)\{1 - \omega[1 - \theta(1 - \rho_s)]^{-1}\}$ percent negative shock to the elasticity $\hat{\xi}_t$. 
To better grasp the role played by deep habits in banking for optimal policy design, it is useful to compare the targeting rule implied by our model (DH) with those one would obtain in a benchmark NK setup and in the standard cost channel model (CC). As we show in full details in Appendix A.1, for given future expectations, the targeting rule can be written as follows:\footnote{For both the NK and the CC models, the targeting rule’s expressions implied by (55) are exact. For the DH model instead, the targeting rule (55) also includes a term depending on expected future inflation. See the Appendix for the details, as well as for the definitions of the composite parameters entering $\Psi_{DH}$.}

$$\hat{\pi}_t = -\psi \Psi_k^{-1} \tilde{y}_t^g, \quad \text{for } k = NK, CC, DH,$$

for

$$\Psi_{NK} \equiv \kappa (\sigma + \varphi),$$

$$\Psi_{CC} \equiv \{\kappa (\sigma + \varphi) - \kappa \eta \sigma\},$$

$$\Psi_{DH} \equiv \{\kappa (\sigma + \varphi) - \kappa \eta (\sigma + \Theta)\},$$

where $\Theta$ is a positive composite parameter defined in equation (A13) in the Appendix, and $\eta \equiv \alpha \bar{\mu}^R / [(1 - \alpha) \beta + \alpha \bar{\mu}^R]$ as previously defined. These analytical expressions allow us to highlight the following results. First, by the definition of $\psi$ and $\Psi_{NK}$, we have that $\psi \Psi_{NK}^{-1} = \epsilon^{-1}$. As $\epsilon > 1$, the latter implies that, in benchmark NK setting, under the optimal policy, inflation is always less volatile than the output gap. Second, since $\eta \in (0, 1]$, we also have that $\Psi_{CC} < \Psi_{NK}$. This inequality means that, for given output gap volatility, the cost channel model features higher inflation volatility with
respect to the benchmark NK model. As already pointed out by Ravenna and Walsh (2006), the credit channel makes it harder to stabilize prices as the central bank has to move the policy rate to counteract the (supply-side) shocks to TFP. In addition to that, inflation can be optimally more volatile than output—that is, $\psi \Psi_{CC}^{-1} > 1$—if $\eta > (\epsilon - 1)/\epsilon$, which occurs if $\alpha$ is sufficiently large.\footnote{Notice that this is always the case if $\alpha = 1$ (100\% working capital requirement) since, in that case, $\eta = 1$.} Third, as shown in the Appendix, we also have that $\Psi_{DH} < \Psi_{CC} < \Psi_{NK}$. According to this last inequality, deep habits in credit market imply a harsher output gap-inflation stabilization trade-off under the optimal policy, which leads to a more significant departure from full price stability.\footnote{It is possible to show numerically that $\Psi_{DH}$ is strictly decreasing in $\theta$. As previously discussed, this result is due to the fact that stronger habits enlarge the discrepancy between the efficiency gap (which the central bank would like to eliminate) and the flexible price gap (which the central bank could eliminate with strict inflation targeting [SIT]).}

6.2 The Case of Commitment

Under commitment, the policymaker announces and implements the optimal state-contingent Ramsey plan that maximizes aggregate welfare, taking into account its direct impact on individual expectations. This allows the policymaker to attain a better trade-off between stabilizing inflation and the output gap in the face of any shock-affecting price setting by individual firms. Figure 3 displays the results in response to a TFP shock for alternative parameterizations, as for the case of discretion displayed in Figure 1.
As for the case of discretion, deep habits exacerbate the inflation-output gap stabilization trade-off faced by the central bank. The optimal response of the economy to the technology shock is a positive output gap, and an initial decline followed by an increase in inflation. The mechanism that leads to larger deviations from full efficiency for a positive $\theta$ is again related to the countercyclicality of credit spreads generated by deep habits. As the figure shows, increasing the steady-state markup in credit markets amplifies the impact of a TFP shock, bringing the economy further away from full stabilization of inflation and the output gap. As expected, conducting monetary policy under commitment achieves a better stabilization trade-off with respect to the case of discretion, for all parameterization considered in Figures 1 and 3. This allows the economy—in particular inflation and the output gap—to remain closer to the efficient (steady state) allocation.

The impulse response functions in response to a shock to credit spreads under commitment are presented in Figure 4. In this case, larger spreads introduced through either deep habits and/or smaller elasticity of the demand for credit do not change the results significantly. Under commitment, fluctuations are contained by roughly the same amount across all three parameterization.

### 6.3 Welfare Analysis

In this section, we evaluate the quantitative importance of deep habits in banking for what concerns the welfare costs associated with (i) the policymaker’s inability to commit to the optimal Ramsey plan; and (ii) the adoption of simpler but suboptimal policy rules. In both cases, we compute the welfare cost both in terms of consumption equivalent (CE) and inflation equivalent (IE) variations.

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**Fig. 4.** Impulse Responses to Credit Spread Shock for Optimal Monetary Policy under Commitment.
**Table 1**

**Welfare Analysis: Discretion versus Commitment**

<table>
<thead>
<tr>
<th></th>
<th>Optimal relative volatility: (SD(\pi)/SD(y^*))</th>
<th>Welfare cost of discretion w.r.t. commitment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CE variation (%)</td>
<td>IE variation (%)</td>
</tr>
<tr>
<td>A. Benchmark NK model</td>
<td>0.16</td>
<td>2.22E-014</td>
</tr>
<tr>
<td>B. Credit channel model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) (\theta = 0)</td>
<td>1.5</td>
<td>0.0095</td>
</tr>
<tr>
<td>(ii) (\theta = 0.25)</td>
<td>1.56</td>
<td>0.0108</td>
</tr>
<tr>
<td>(b) 2% spread</td>
<td>1.63</td>
<td>0.0122</td>
</tr>
<tr>
<td>(iii) (\theta = 0.5)</td>
<td>1.76</td>
<td>0.015</td>
</tr>
<tr>
<td>(a) 2% spread</td>
<td>1.91</td>
<td>0.019</td>
</tr>
<tr>
<td>(b) 4% spread</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With respect to CE, we follow Schmitt-Grohé and Uribe (2007) and define the welfare cost of adopting an alternative policy \(A\) with respect to a reference policy \(R\) as the fraction \(\nu\) of the consumption path under policy \(R\) that must be given up to make the household as well off under policy \(A\) as under policy \(R\). In our case, policy \(R\) is commitment and policy \(A\) is discretion. With respect to IE, we follow Dennis and Soderstrom (2006) by computing the permanent decrease in annual inflation needed to compensate the household for a switch from commitment to discretion.

**Discretion versus commitment.** Since the seminal work of Kydland and Prescott (1977), it is well known that policy actions taken under commitment deliver higher welfare than those under discretion. As private agents’ decisions depend on future expectations, by announcing credible policy plans, a committed government can strategically manipulate expectations and attain a more favorable policy trade-off between stabilizing the output gap and inflation. The impulse response analysis presented in the previous section highlights the possibility of even higher welfare gains if deep habits in credit markets are at work.

Table 1 presents the results of our analysis for the benchmark NK model (where the credit channel is absent) and alternative parameterizations of the credit channel model (differing in the extent of deep habits and the steady-state size of the credit spread). The second column reports the optimal relative volatility (ORV)—that is, the standard deviation of inflation relative to that of the output gap occurring under the optimal discretionary policy—while the third and fourth columns report the CE and IE welfare cost of acting under discretion (with respect to commitment). As already stressed toward the end of Section 6.1, in the benchmark setting, optimal monetary policy makes inflation less volatile than the output gap. For this simple case, the relative standard deviation is, in fact, equal to \(e^{-1}\), which equals 0.16 under our calibration.

It also appears that the welfare costs of acting under discretion are quite negligible, both in CE and IE terms. This is clearly not the case once a credit channel and deep habits in banking are considered. In particular, the following results emerge. First, the credit channel makes inflation more volatile than output. For the case of \(\theta = 0\),...
the ORV is equal to \( \epsilon^{-1}(\sigma + \varphi)/[\sigma(1 - \eta) + \varphi] \), which reduces to \( (\sigma + \varphi)/\epsilon \varphi = 1.5 \) under our calibration (recall that \( \eta = 1 \) if \( \alpha = 1 \)). This pattern is reinforced by our banking friction: if deep habits and/or the credit spread are strengthened, the ORV of inflation to output increases. This finding is consistent with the analytical result presented of equation (55), where, for given output gap volatility, the departure from price stability is larger, the stronger the banking friction.

Second, the welfare costs of acting under discretion (with respect to commitment) are strictly increasing in the degree of deep habits and the size of the credit spread. For instance, if \( \theta = 0.5 \) (B.iii. in the table), the CE costs are 50% larger than those occurring without deep habits with a 2% spread, and twice as large with a 4% spread. This effect is also evident if measured in terms of permanent inflation. For \( \theta = 0.5 \) and a 2% spread, the household would require almost a 1.6% permanent decrease in yearly inflation in order to accept a switch from full commitment to discretion. This value goes up to almost 1.8% if the credit spread becomes 4%.

Figure 5 summarizes the main findings of the welfare analysis. The left-hand panel displays the ORV (under discretion) as a strictly increasing function of \( \theta \). While the same pattern remains, a higher average credit spread significantly increases the ORV for any given \( \theta \). The right-hand panel displays instead the CE welfare costs of setting policy under discretion. Similarly to the ORV, welfare costs appear to be strictly increasing in \( \theta \), with the size of the average credit spread acting as a upward shifter.

Why are deep habits hampering the welfare costs of acting under discretion? The reason has to do once again with the management of expectations. Commitment is superior to discretion simply because the policymaker internalizes how its policy plan affects future private expectations that, in turn, affect the equilibrium relationships which the policymaker himself is optimizing with respect to. From equation (45), we can see that, with deep habits, the key distortion (the credit spread) is driven not only by current economic conditions—for example, current policy rates, current economic activity (loan demand)—but also (and most importantly) by market expectations.
on future values for loans demand, policy rates, inflation, and the spread itself. A discretionary policymaker taking those expectations as given is less capable of shielding the economy from exogenous shocks that, through their impact on the credit spread, will destabilize inflation and the output gap. By announcing a credible state-contingent Ramsey plan, a committed policymaker can shape those expectations, and to a certain extent indirectly control fluctuations in credit spreads. With inflation and output closer to targets, welfare is higher than under discretion. As deep habits amplify the expectation channel on credit spreads, the welfare gains of commitment are a strictly increasing function of $\theta$.

Suboptimal policy rules. Optimal monetary policy requires the monetary authority to have full knowledge of the economy’s underlying structural relationships; otherwise, it would not be able to attain the desired inflation-output stabilization. It is therefore interesting to consider the welfare consequences of adopting suboptimal policy rules which do not require full information on behalf of the monetary authority. More specifically, we compute the CE variation welfare costs (with respect to the truly optimal policy under commitment) for the following alternative policy rules: strict inflation targeting, SIT (i.e., the nominal interest rate is moved in order to keep inflation at its target at all times, $\hat{\pi}_t = 0$); flexible inflation targeting, FIT (i.e., the inflation-output gap targeting rule that the monetary authority would implement in the benchmark NK model without credit frictions, $\hat{\pi}_t = -\psi_0(\Delta^S\hat{y}_t)$); a standard Taylor rule, TR$_1$ (i.e., the nominal interest rate $\hat{r}_t$ is set according to the rule $\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t$, with $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$, as proposed by Taylor [1993]); a spread-augmented Taylor rule, TR$_2$ (i.e., the policy rate responds negatively to the credit spread, $\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t - \phi_\mu \hat{\mu}_t$, with $\phi_\mu = 0.25$); and a credit-augmented Taylor rule, TR$_3$ (i.e., the policy rate responds positively to loans, $\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi_l \hat{l}_t$, with $\phi_l = 0.25$). Notice that both Taylor rules are expressed only in terms of observables. Namely, it is assumed that the monetary authority responds to the output level and not the output gap, as the latter is hardly observed with precision by the policymaker. The negative and positive sign restrictions imposed, respectively, on $\phi_\mu$ and $\phi_l$ follow the common wisdom in the related literature.\[36\] The results are reported in Table 2.

Although it provides a good approximation to the truly optimal policy in the benchmark NK model, SIT always leads to equilibrium indeterminacy (hence, sunspot-driven fluctuations) in a model with a credit channel, independently from the extent of deep habits. While its quantitative performance is clearly superior to SIT and all Taylor rule specifications considered, FIT displays welfare costs that are strictly increasing in deep habits. For instance, they become about three times larger if we move from the simple zero-spread cost channel model (line B.i.) to a deep habits model with $\theta = 0.5$ and a 4% yearly spread (line B.ii.b).

Credit frictions also make less desirable to follow a standard Taylor rule (TR$_1$). Its welfare costs for the case of $\theta = 0.5$ and a 4% yearly spread (line B.ii.b) are

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36. See, for instance, Curdia and Woodford (2010) and Christiano, Motto, and Rostagno (2010).
more than 30 times larger than for the benchmark NK model (line A). Nevertheless, the analysis shows that there can be some welfare gains from letting the policy rate respond to movements in credit demand when countercyclical spreads are present (compare TR\(_1\) with TR\(_3\) for the case of \(\theta = 0.5\), under both a 2% and 4% spread). On the contrary, there appear to be no welfare gains (actually, even losses) from responding to the observed credit spread (compare TR\(_1\) with TR\(_2\)).

A possible interpretation of the apparently conflicting results given by rules TR\(_2\) and TR\(_3\) is the following. Consider the spread-augmented rule TR\(_2\), \(\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t - \phi\mu \hat{\mu}_t\). Using the definition \(\hat{\mu}_t = \hat{L}_t - \hat{r}_t\), such rule is equivalent to \(\hat{r}_t = (1 + \phi\mu)^{-1}(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t - \phi\mu \hat{L}_t)\). An increase in \(\phi\mu\) has then two opposite effects. On the one hand, it is beneficial: following an increase in the loan rate, it leads to a lower policy rate, and hence a lower marginal cost for banks, which, in turn, provides a lower incentive to raise loan rates. On the other hand, it is detrimental as it implicitly determines a milder response to both inflation and output, which, as well known, could potentially lead to larger aggregate fluctuations. It appears that these two effects cancel out each other in our quantitative analysis. Consider now the credit-augmented rule TR\(_3\), \(\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi\ell \hat{\ell}_t\). Because of the countercyclicality generated by deep habits, a positive \(\phi\ell\) is equivalent to an implicit negative response of the policy rate to the credit spread (as the latter decreases when loan demand is higher). However, differently from the rule TR\(_2\), the rule TR\(_3\) does so without inducing a milder response to inflation and/or output.

7. RELATION WITH EXISTING LITERATURE

Our work feeds into the literature started by Ravenna and Walsh (2006) on the macrofinancial linkages and their implications for the conduct of monetary policy.
Ravenna and Walsh were the first to endogenize the credit/cost channel of monetary policy by introducing an endogenous cost-push shock to the Phillips curve. With firms borrowing to pay for working capital, the interest rate on loans enters the Phillips curve, thus counteracting the standard aggregate demand channel. Thus, the cost channel breaks the “divine coincidence” and generates a meaningful trade-off between stabilizing inflation and the output gap in response to demand-side shocks, as well as it mitigates (amplifies) the reaction of prices (output) to monetary policy shocks.

Within this literature, the paper that is most closely related to ours is Aksoy, Basso, and Coto-Martinez (2013) who also study monetary policy and lending relationships by introducing the deep habits in credit markets as in Aliaga-Díaz and Olivero (2010a). They show that spread movements are crucial for policy even when a standard Taylor rule is employed, and that strong credit relationships may lead to indeterminacy of equilibrium that forces the central bank to react to changes in credit conditions. Aksoy, Basso, and Coto-Martinez (2013) explore several alternative interest rate rules. However, since they do not study optimal policy, their work cannot speak to the welfare properties of price stabilization, the stabilization trade-off facing central banks, or the gains of commitment.

Most of the work that followed can be classified in three main strands. The first line of work focuses on the implications for monetary policy of informational asymmetries in credit markets and the implied need for loan monitoring. Aikman and Paustian (2006), Carlstrom, Fuerst, and Paustian (2010), Christiano, Motto, and Rostagno (2010), Williamson (2012), Agenor, Bratsioitis, and Pfajfar (2014), and De Fiore and Tristani (2012) all model firms that need to externally finance their working capital, and due to asymmetric information, they need to pledge their net worth as collateral for these loans. This friction leads banks to optimally charge an external finance premium above the policy rate, which, in turn, enters as an endogenous cost-push shock in the Phillips curve.37 With this environment, they obtain results qualitatively similar to ours: optimal monetary policy deviates from perfect price stability. Aikman and Paustian also conclude that responding to asset prices or credit growth as additional targets through the interest rate rule is detrimental to welfare relative to a policy of strict price stability.38

In the second strand, banks are required to operate a costly function for the production or management of loans, as in Goodfriend and McCallum (2007) and Cúrdia and Woodford (2009). With the marginal cost of loan production being procyclical, this literature introduces some degree of procyclicality to the margin between loan and interbank rates. The third line of work introduces staggered loan contracts that determine heterogeneously sticky interest rates and an empirically plausible incomplete pass-through from policy rates to lending rates (see Teranishi

37. While in De Fiore and Tristani (2012) what firms can use as collateral is an exogenous endowment, in Carlstrom et al. (2010), it is endogenous, which allows them to introduce some additional feedback between endogenous net worth and asset prices. In Carlstrom et al. (2010), a monetary policy response to supply-side shocks affects share prices and the market value of net worth and, through the collateral constraint, interest rates and the cost of labor. Under some conditions, optimal monetary policy still consists of STT.

38. De Fiore and Tristani (2012) can reproduce countercyclical premia only when allowing for a set of additional shocks.
This allows for an accelerator effect through which economic fluctuations become more persistent and of greater amplitude.

Our contribution relative to these papers is threefold. First, we show that customer-market features in banking can have significant implications for monetary policy as they generate countercyclical credit spreads, even in the absence of default risk. Second, by endogenizing the incomplete pass-through from policy to lending rates, our framework is consistent with the empirical evidence on the sluggish adjustment of loan rates in response to shocks to open-market rates presented by Slovin and Sushka (1983) and Berger and Udell (1992). Third, in contrast to several of these works that mainly focus on monetary policy conducted through exogenous interest rate rules, we study optimal policy and the welfare costs of using suboptimal rules.

Also related are a series of papers on the aggregate consequences for monetary and fiscal policy of “deep habits” in goods markets. Ravn et al. (2010), Leith, Moldovan, and Rossi (2012), and Givens (2016) focus on monetary policy and Zubairy (2014a, 2014b) on fiscal. This work echoes our results, but with deep habits concentrated in the retail sector rather than in credit markets.

8. CONCLUSIONS

We have augmented a small-scale NK DSGE framework with two frictions in the banking sector: monopolistic competition and features of a customer-market type of model. We have modeled the latter as in the deep habits in credit markets model of Aliaga-Díaz and Olivero (2010a), by assuming that the liquidity needs of borrowing-constrained firms are served by a loan composite that depends on the amount of past bank-specific loans. This feature captures, in reduced form, the documented "hold-up" effect and the existence of switching costs in banking relationships. By making interest rate spreads between loans and deposits countercyclical as in the data, it implies that during a phase of economic expansion, banks might find optimal to lower current lending rates to greatly expand their customer base, which will then be locked into a long-term relationship. We have then used this framework to study the conduct of optimal monetary policy, as well as the welfare costs of the lack of commitment and of alternative suboptimal policy rules.

Our analysis shows that the combination of monopolistic competition and deep habits in credit markets exacerbates the trade-off between stabilizing inflation and the

39. Ravn et al. (2010) show that deep habits make firms set prices in a forward-looking manner (even under flexible prices) and are therefore complementary to standard ways of introducing nominal price rigidities. Thus, they can account for the inflation persistence puzzle without relying on unreasonable extents of nominal rigidities.

40. They show that under deep habits, the policymaker optimally allows for a larger output gap and the economy slips further away from the flexible price efficient allocation. They also find that, relative to superficial habits, deep habits exacerbate the welfare cost of deviating from the optimal policy under commitment.

41. Givens (2016) shows that deep habits in consumption weaken the stabilization trade-offs facing a discretionary planner.
output gap in optimal monetary policymaking, under both discretion and commitment. In particular, because of its impact on the credit-related component of marginal costs, the nominal interest rate cannot be used to fully insulate the economy from shocks to aggregate productivity. After a positive technological shock, optimal policy prescribes a positive output gap, a deflation, and a drop in the policy rate that are all larger than in the credit channel NK model without deep habits. This result hinges on the countercyclicality of credit spreads generated by deep habits: as the monetary authority induces a positive output gap (via a cut in the policy rate) to contain deflation, the credit spread falls, putting further downward pressure on marginal costs, which, in turn, leads to a more severe deflation.

From a quantitative perspective, the departure from price stability under the optimal policy is substantially larger than that implied by the simple cost channel model of Ravenna and Walsh (2006). For our benchmark calibration of deep habits, the deviation of inflation from its target under the discretionary regime can be 40–60% larger, depending on the degree of market power in banking. The welfare costs of deviating from the optimal Ramsey plan are higher than in the standard cost channel model by 100% in terms of the CE variation and 40% in terms of the inflation-equivalent variation.

The welfare costs of committing to simpler but suboptimal policy rules also appear to be quite sizable and to be strictly increasing in the degree of deep habits in banking. Taking into account that the welfare gains from commitment in a benchmark NK model without credit market imperfections are typically very small, this result highlights the quantitative importance of optimal monetary policy commitment when there are imperfections in financial intermediation.

Working on this paper, we were able to uncover the fact that countercyclical credit spreads can become a source of equilibrium indeterminacy in the sense that even when satisfying the Taylor principle, simple instrumental interest rate rules can induce nonfundamental sunspot-driven fluctuations. This result casts some doubts on the desirability of feedback rules to implement the optimal policy plan. Studying this indeterminacy feature in depth is left for future work.

To conclude, in our framework, banks are not subject to any type of macroprudential regulations. Understanding how the introduction of these regulations interacts with optimal monetary policymaking is worth of further efforts in the literature.

APPENDIX A: OPTIMAL MONETARY POLICY

A.1 Some Analytics

The optimal monetary policy problem can be expressed as the minimization of the following Lagrangian expression:

$$\min L_0 = \min E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left[ \hat{\pi}_t^2 + \psi \left( \hat{y}_g^p \right)^2 \right] + \lambda_{1,t} \left[ \beta \hat{\pi}_{t+1} + \kappa (\sigma + \varphi) \hat{y}_t^\varphi + \kappa \eta \hat{r}_t^L + \kappa \eta \hat{r}_t^K - \hat{\pi}_t \right] \right\}$$
\[ \begin{align*}
&\lambda_2, t \left[ \hat{y}_t^{g} - \frac{1}{\sigma} \hat{r}_t + \frac{1}{\sigma} E_t \hat{a}_{t+1} + \hat{u}_t - \hat{y}_t^{g} \right] \\
&\lambda_3, t \left[ A_r \hat{r}_t - A_\pi \hat{a}_{t+1} - A_y' \hat{y}_{t+1} - A_\mu' \hat{\mu}_{t+1} \\
&+ A_y' \hat{y}_{t+1} + A_s' \hat{s}_t - A_y \hat{y}_{t+1} + A_s \hat{s}_{t-1} \\
&+ (1 - \theta) \chi A_u \hat{a}_t - \omega \xi_t - A_\mu \hat{\mu}_t \right] \\
&\lambda_4, t \left[ \bar{\rho}_t \hat{s}_{t-1} + (1 - \rho_s) (1 - \theta) (1 + \sigma + \varphi) \hat{y}_t^{g} \\
&+ (1 - \rho_s) (1 - \theta) \chi \hat{a}_t - \hat{s}_t \right] \\
\end{align*} \]

where \( \hat{u}_t \equiv [(1 + \varphi)/(\sigma + \varphi)](E_t \hat{a}_{t+1} - \hat{a}_t) \), \( \bar{\rho}_s \equiv \rho_s + (1 - \rho_s) \theta \), and, for notational purposes, we have defined the following composite structural parameters that multiply the variables in equation (45):

\[ \begin{align*}
A_\pi &= A'_r = A'_\mu \equiv \frac{\theta \beta \omega (1 - \rho_s)}{1 - \beta \theta (1 - \rho_s)}, \\
A'_y &\equiv \theta (1 + \sigma + \varphi) A_\pi, \\
A'_s &\equiv \frac{\theta \omega}{1 - \beta \theta (1 - \rho_s)}, \\
A_u &\equiv \frac{\rho \theta^2 \beta \omega (1 - \rho_s) - \theta \omega}{(1 - \theta) [1 - \beta \theta (1 - \rho_s)]}, \\
A_\mu &\equiv 1 - \frac{\omega}{1 - \beta \theta (1 - \rho_s)}. \\
\end{align*} \]

First-order conditions with respect to \( \hat{a}_t, \hat{y}_t^{g}, \hat{r}_t, \hat{\mu}_t^R \), and \( \hat{s}_t \) give the following equations:

- for \( t = 0 \):
  \[ \hat{a}_t = \lambda_{1,t} \]  \hspace{1cm} (A1)
  \[ \psi \hat{y}_t^{g} = -\kappa (\sigma + \varphi) \lambda_{1,t} + \lambda_{2,t} + A_y \lambda_{3,t} + (1 - \rho_s) (1 - \theta) (1 + \sigma + \varphi) \lambda_{4,t} \]  \hspace{1cm} (A2)
  \[ \frac{1}{\sigma} \lambda_{2,t} = \kappa \eta \lambda_{1,t} + A_r \lambda_{3,t} \]  \hspace{1cm} (A3)
  \[ \kappa \eta \lambda_{1,t} = A_\mu \lambda_{3,t} \]  \hspace{1cm} (A4)
  \[ \lambda_{4,t} = \bar{\rho}_s \beta E_t \lambda_{4,t+1} + A'_y \lambda_{3,t} + \beta A_s E_t \lambda_{3,t+1} \]  \hspace{1cm} (A5)

- for \( t \geq 1 \):
  \[ \hat{a}_t = \lambda_{1,t} - \lambda_{1,t-1} - \frac{1}{\sigma \beta} \lambda_{2,t-1} + \frac{A_\pi}{\beta} \lambda_{3,t-1} \]  \hspace{1cm} (A6)
\[
\psi \hat{y}_t^g = -\kappa (\sigma + \varphi) \lambda_{1,t} + \lambda_{2,t} + A_y \lambda_{3,t} - \frac{\lambda_{2,t-1}}{\beta} - \frac{A'_s}{\beta} \lambda_{3,t-1} + (1 - \rho_s)(1 - \theta)(1 + \sigma + \varphi) \lambda_{4,t}, \quad (A7)
\]

\[
\frac{1}{\sigma} \lambda_{2,t} = \kappa \eta \lambda_{1,t} + A_y \lambda_{3,t} - \frac{A'_s}{\beta} \lambda_{3,t-1}, \quad (A8)
\]

\[
\kappa \eta \lambda_{1,t} = A_\mu \lambda_{3,t} + \frac{A'_s}{\beta} \lambda_{3,t-1}, \quad (A9)
\]

\[
\lambda_{4,t} = \tilde{\rho}_s \beta E_t \hat{\lambda}_{4,t+1} + A'_y \lambda_{3,t} + \beta A_s E_t \lambda_{3,t+1}. \quad (A10)
\]

As pointed out by McCallum and Nelson (2004), one could conceive the optimal solution under discretion in the following manner. The policymaker implements (A1)–(A5) in period 0—where \( \lambda_{j,t-1} = 0 \) for \( j = 1, 2, 3 \) since there are no past promises—and plans to implement (A6)–(A10) in each subsequent period. However, when period 1 arrives, the discretionary government resolves the optimal policy problem and implements (A1)–(A5) for \( t = 1, 2, \ldots \).

After substituting out the Lagrange multipliers in the system (A1)–(A5), simple algebra allows us to get the following relationship:

\[
\psi \hat{y}_t^g = - \left\{ \kappa (\sigma + \varphi) - \kappa \eta \left[ \sigma \left( 1 + \frac{A'_s}{A_\mu} \right) + \frac{A'_y}{A_\mu} \right] \right\} - \delta \lambda_{4,t}, \quad (A11)
\]

where \( \delta \equiv (1 - \rho_s)(1 - \theta)(1 + \sigma + \varphi) \). Next, consider equation (A5), which regulates the equilibrium dynamics of the multiplier \( \lambda_{4,t} \) and let \( \pi_t \equiv A'_s \lambda_{3,t} + \beta A_s E_t \lambda_{3,t+1} \). Since we are restricting to a stationary solution, assume that \( \pi_t \) follows a simpler AR(1) process with persistence parameter \( \rho_\pi \in (0, 1) \) and a mean-zero independent and identically distributed disturbance. Given that \( \tilde{\rho}_s \beta \in (0, 1) \), equation (A5) can be solved forward to give the following solution: \( \lambda_{4,t} = (1 - \rho_\pi \tilde{\rho}_s \beta)^{-1} \pi_t \). From the system (A1)–(A5), we also have that \( \lambda_{3,t} = (\kappa \eta / A_\mu) \pi_t \), such that:

\[
\lambda_{4,t} = \frac{\kappa \eta A'_s}{A_\mu (1 - \rho_\pi \tilde{\rho}_s \beta)} \hat{\pi}_t + \frac{\kappa \eta \beta A_s}{A_\mu (1 - \rho_\pi \tilde{\rho}_s \beta)} E_t \hat{\pi}_{t+1}. \quad (A12)
\]

This expression for \( \lambda_{4,t} \) can then be substituted into (A11) to give the following relationship:

\[
\psi \hat{y}_t^g = - \left\{ \kappa (\sigma + \varphi) - \kappa \eta (\sigma + \Theta) \right\} \hat{\pi}_t - \delta \frac{\kappa \eta \beta A_s}{A_\mu (1 - \rho_\pi \tilde{\rho}_s \beta)} E_t \hat{\pi}_{t+1}, \quad (A13)
\]

where \( \Theta \equiv \sigma (A'_r / A_\mu) + (A'_y / A_\mu) + \delta \theta (A''_r / A_\mu)(1 - \rho_\pi \tilde{\rho}_s \beta)^{-1} \). Equation (A13) corresponds to what Svensson and Woodford (2005) refer to as a targeting rule: it

42. This concept of optimal monetary policy under discretion is different from the Markov perfect equilibrium concept we have used in Section 6.1. However, it proves useful to derive (to a good approximation) the analytics behind the optimal policy plan. See McCallum and Nelson (2004) for a discussion/comparison of the two approaches.
defines the optimal output-gap inflation volatility trade-off faced by the policymaker. To grasp how deep habits in banking affect this trade-off, we define the following composite coefficients:

\[
\Psi_{NK} \equiv \kappa (\sigma + \varphi), \quad \Psi_{CC} \equiv \{\kappa (\sigma + \varphi) - \kappa \eta \sigma\}, \quad \Psi_{DH} \equiv \{\kappa (\sigma + \varphi) - \kappa \eta (\sigma + \Theta)\}.
\]

They correspond to the coefficient multiplying inflation in (A13) in, respectively, the benchmark NK model, the cost channel model of Ravenna and Walsh (2006), and our model with deep habits in banking. Equation (A13) implies that, for given \(E_t \hat{\pi}_{t+1}\) and given volatility for the output gap, stabilizing inflation is more costly (i.e., inflation volatility would be larger in equilibrium), the smaller is \(\Psi\). For instance, since \(\Psi_{CC} < \Psi_{NK}\), it immediately follows that the cost channel model of Ravenna and Walsh features more inflation volatility with respect to the benchmark NK model.

Consider now the case of deep habits, for which \(\Psi_{DH}\) is the relevant coefficient. Notice that \(A_r, A_y, \text{ and } A_\pi\) are all strictly positive, and so are \(\delta\) and \((1 - \rho_x \bar{p}_t \beta)\), while simple (but tedious) algebra shows that, under Assumption 1, \(A_{\mu} > 0\) as well. It then follows that \(\Theta > 0\) and therefore \(\Psi_{DH} < \Psi_{CC}\): that is, for given volatility of the output gap, equilibrium inflation is more volatile in a model with deep habits in credit markets.

### A.2 Welfare Cost Computation

**Consumption equivalent variation.** We follow Schmitt-Grohé and Uribe (2007) and define the welfare cost of adopting an alternative policy \(A\) with respect to a reference policy \(R\) as the fraction \(\nu\) of the consumption path under policy \(R\) that must be given up to make the household as well off under policy \(A\) as under policy \(R\). More specifically, \(\nu\) is computed as the unique solution to the following equality:

\[
E \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t^A)^{1-\sigma}}{1-\sigma} - \frac{(H_t^A)^{1+\psi}}{1+\psi} \right] = E \sum_{t=0}^{\infty} \beta^t \left[ \frac{((1-\nu)C_t^R)^{1-\sigma}}{1-\sigma} - \frac{(H_t^R)^{1+\psi}}{1+\psi} \right].
\]

The computational procedure involves following steps. Let \(R\) strand for “optimal policy under commitment” and \(A\) for “optimal policy under discretion.”
Step 1. We compute the value of aggregate welfare under policy $J$, for $J = A, R$, using the second-order approximation procedure described by Schmitt-Grohe and Uribe for the computation of unconditional welfare:

$$U^J \equiv \hat{U} - \frac{\epsilon \hat{C}^{1-\sigma} \kappa}{2(1 - \beta)} \left[ Var \left( \hat{\pi}_J, t \right) + \psi Var \left( \hat{y}_g^J, t \right) \right],$$

(A14)

where $\hat{U} \equiv (1 - \beta)^{-1} [(1 - \sigma)^{-1} \hat{C}^{1-\sigma} - (1 + \varphi)^{-1} \hat{H}^{1+\varphi}]$.

Step 2. For $J = A, R$, we compute the percentage reduction in steady-state consumption, $\upsilon_J$, that would make the representative agent indifferent between the efficient steady-state allocation and the allocation occurring under policy regime $J$. Given the value of $U^J$ computed in Step 1, we solve the following equation with respect to $\upsilon_J$:

$$U^J = \frac{1}{1 - \beta} \left[ \hat{C} (1 - \upsilon_J)^{1-\sigma} \right] - \frac{\hat{H}^{1+\varphi}}{1 + \varphi}.$$  

(A15)

Letting $\Upsilon_J \equiv (1 - \beta)U^J + (1 + \varphi)^{-1} \hat{H}^{1+\varphi}$ and $\kappa_J \equiv (1 - \sigma)\Upsilon_J^{1-\sigma}$, simple algebra gives that $\upsilon_J = 1 - (\kappa_J / \hat{C})$.

Step 3. The consumption-based welfare cost associated with adopting the alternative policy $A$ with respect to the reference policy $R$ is then $\upsilon \equiv \upsilon_A - \upsilon_R$.

Inflation equivalent variation. We follow Dennis and Soderstrom (2006), and a more recent application by Demirel (2013), by computing the permanent decrease in yearly inflation needed to compensate the household for a switch from commitment to discretion. For given arbitrary paths of inflation and the output gap, $\{\hat{\pi}_t, \hat{y}_g^t\}_{t=0}^{\infty}$, we have that (a second-order approximation to) aggregate welfare, conditional on information available at $t = 0$, is equal to:

$$W_0 = \frac{\hat{U}}{1 - \beta} - \frac{\epsilon \hat{C}^{1-\sigma} \kappa}{2(1 - \beta)} E_0 \sum_0^{\infty} \beta^t \left[ \hat{\pi}_t^2 + \psi (\hat{y}_g^t)^2 \right].$$

(A16)

Given (A16), the welfare gain from a $x\%$ permanent decrease in quarterly inflation is then equal to $[(\epsilon \hat{C}^{1-\sigma}) / 2(1 - \beta)\kappa] (x/100)^2$.

For the computation of $x$, we use Monte-Carlo simulation methods. Let $N$ be the number of simulations (we set $N = 10,000$). Then, for $n = 1 : N$, we proceed as follows.

Step 1. We simulate time series for inflation and the output gap under both discretion and commitment. We then compute the values of (A16), under both regimes, by truncating the infinite summation at a very large $T$. We denote the respective values by $W_{0,n}^{disc}$ and $W_{0,n}^{com}$, where the subscript $n$ denotes the specific simulation.

Step 2. Since $W_{0,n}^{com} > W_{0,n}^{disc}$, we compute the percentage decrease in inflation that is required to increase welfare by $\Delta_n \equiv W_{0,n}^{com} - W_{0,n}^{disc}$ by solving the
following equation:
\[
\frac{\epsilon C^{1-\sigma}}{2 (1 - \beta) \kappa} \left( \frac{x_n}{100} \right)^2 = \Delta_n.
\]

**Step 3.** Given the series \( x_n \), for \( n = 1 : N \), we compute its sample mean and multiply it by 4 to obtain annual values: \( x = 4 \times \sum_{n=1}^{N} \frac{x_n}{N} \).

**LITERATURE CITED**


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